Lets count!

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October 27, 2011

1 Review

Here is a list of counting problems. Use common sense to solve the problems. We shall review these questions, group them together and try to formulate some principles that will aid us solve more complicated coutning problems.

We start with the 40 students in our class.

1. Our class meets for Hanh's birthday party. They all hug each other once, how many hugs took place?

2. Linh says that he has 11 friends among the 40 students. Ky says he has only 9. Phuong says: "I bet you that there are two students that have the same number of friends". Prove that Phuong is right.

3. To celebrate, the class decided to sit around a table. Hoang suggested that for fun, they shall change the sitting every 20 seconds. Thành suggested that since it is Hanh's birthday, she should remain seated at the head of the table. How long will the party last if they are determined to go through all possible different sitting arrangements?

4. To continue celebrating the birthday, the class decided to do a folk dance. The dance requires 3 persons. In how many ways can we form 10 quadruples of students from the 40 students?

5. Trung just returned from Bad Trang. He saw there a a ceramic square divided into 4 triangles by its two diagonals. There were 10 cans of paint, each with a different color. He was told that each triangle needs to be colored in a different color. Trung wondered, how many distinct tiles can be produced? Can you help him?

6. Thinh thought about it, then recalled the Rubik Cube. He wondered, suppose you take a cube, it has six faces. Suppose you want to color the faces with six different

colors, Red, Blue, White, Yellow, Green and Black. How many different cubes can you produce?

7. Chi suggested to do something especially nice. He suggested that every student should donate 2000 VND, go out, find poor kids and give them the money. The class quickly collected 80,000 VND. They went ouside and quickly found 12 kids. In how many different ways could the money be distributed (assuming that there were 27 2000 VND, so every kid gets a multiple of 2000 Dongs).

8. Dung suggested what it would be nice to have 600,000 and give each kid at least 2000 VND. In how many ways can the money be distributed?

9. Back at the party, the students decided to organize a mini chess tournament. Hanh brought out 20 tables and wondered, in how many ways can the 40 students be partitioned into pairs for the chess tournament?

10. For a finale, the students decided to fill lotto tickets and hopefully win a big prize. The lotto ticket has 49 numbers arranged in 7 rows, each row has 7 numbers. A winning ticket has 7 numbers, one number from each row. The students decided that each one will fill 10 tickets. What is the chance that the class will win the lottery?

Counting bit strings. A bit string is a seqence of 0's and 1's.

1. How many different bit strings of length 20 begin with 1 and end with 0?

2. How many different bit strings of length 21 have 10 1's and 11 0's?

3. How many bit strings of length n have $\lfloor \frac{n}{2} \rfloor$ 1's

4. How many bit strings of length n begin with a 1 or end with a 0?

5. How many bit strings of length n do not contain 00 as a substring?

6. How many bit strings of length n contain 00 as a substring?

7. How many bit strings of length n do not contain 101 as a substring?

Misc. Counting

1. In a group of 30 people, 8 speak English, 12 speak Spanish and 10 speak French. It is known that 5 speak English and Spanish, 5 speak Spanish and French, and 7 speak English and French. The number of people speaking all three languages is 3. How many do not speak any of these languages?

2. What is the coefficient of x^{13} in the expansion of $(2x - \frac{3}{x^3})^{20}$

3. What is the coefficient of x^{12} in the expansion of $(2x - \frac{3}{x^3})^{20}$

4. How many integers < 10000 are divisible by 8 and by 11 but not by both? How many integers < 100000 are divisible by 8 by 11 and by 12 but not by the product of any two of them?

5. How many integers < 195 are relatively prime to 195?

6. How many integers < 1729 are relatively prime to 1729?

7. How many integers < 1729 are divisible by 7, 13 or 19 but not by the product of any two of these numbers?