

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 5.4—Binomial Coefficients



— Page references correspond to locations of Extra Examples icons in the textbook.

p.364, icon at Example 2

#1. Write the expansion of $(x + 2y)^3$.

Solution:

By the binomial theorem,

$$\begin{aligned}(x + 2y)^3 &= \binom{3}{0}x^3(2y)^0 + \binom{3}{1}x^2(2y)^1 + \binom{3}{2}x^1(2y)^2 + \binom{3}{3}x^0(2y)^3 \\ &= x^3 + 6x^2y + 12xy^2 + 8y^3.\end{aligned}$$

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#2. Find the coefficient of $a^{17}b^{23}$ in the expansion of $(3a - 7b)^{40}$.

Solution:

We expand $(3a - 7b)^{40}$ using the binomial theorem, locate the term with the product $a^{17}b^{23}$, and then find the coefficient:

$$\begin{aligned}(3a - 7b)^{40} &= \cdots + \binom{40}{17}(3a)^{17}(-7b)^{23} + \cdots \\ &= \cdots + \binom{40}{17}3^{17}(-7)^{23}a^{17}b^{23} + \cdots.\end{aligned}$$

Thus, the coefficient is $\binom{40}{17}3^{17}(-7)^{23}$, which can also be written as $\binom{40}{23}3^{17}(-7)^{23}$.

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#3. Write the expansion of $\left(x^2 - \frac{1}{x}\right)^8$.

Solution:

We use the binomial theorem. We then use various rules for exponents to simplify the terms.

$$\begin{aligned}\left(x^2 - \frac{1}{x}\right)^8 &= \sum_{i=0}^8 \binom{8}{i} (x^2)^i \left(\frac{-1}{x}\right)^{8-i} \\ &= \sum_{i=0}^8 \binom{8}{i} \frac{x^{2i} (-1)^{8-i}}{x^{8-i}} \\ &= \sum_{i=0}^8 \binom{8}{i} x^{3i-8} (-1)^{8-i} \\ &= x^{-8} - 8x^{-5} + 28x^{-2} - 56x^1 + 70x^4 - 56x^7 + 28x^{10} - 8x^{13} + x^{16} \\ &= \frac{1}{x^8} - \frac{8}{x^5} + \frac{28}{x^2} - 56x + 70x^4 - 56x^7 + 28x^{10} - 8x^{13} + x^{16}.\end{aligned}$$
