Discrete Mathematics and Applications

Moshe Rosenfeld

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moishe@u.washington.edu

1 Assignment No.6: Finite sets and permutations

Due: Tuesday, 25 Oct.

Please submit your answer in a neat, readable properly organized format.

2 Finite Sets

1. You are given a list of 35 teams from our class (40 students). Each team has an odd number of students and each two teams have an even number of students in common. Prove that you can add another team with an odd number of students that has an even number of students in common with each of the other teams.

2. What is the equation of the line in PG(7) through the points $(1, 1, 1)$ *and* $(2, 0, 1)$

3. 16 friends meet for dinner in a restaurant. They are seated at four tables, 4 diners per table. Arrange a schedule of five evenings so that each person will dine exactly once with every other person.

3 Permutations

Note: in the Supplements folder you can find a file called permutations-lexicographic-order.pdf with a sample of questions and answers.

1. Express each of the following cycles as a product of transpositions.
   a. (1 2 3 7 9)
   b. $(1\ 2\ 3\ \ldots n)$
   c. $(1\ 2\ 3\ 4) \circ (2\ 3\ 4\ 5) \circ (1\ 2\ 3\ 4)$

2. Let $\pi = 7\ 3\ 1\ 6\ 9\ 5\ 2\ 4\ 8$ $\quad \mu = 7\ 3\ 1\ 4\ 5\ 8\ 6\ 2\ 9$
   a. Which permutation is lexicographycally "smaller?"

b. Which permutation immediately follows π in the lexicographic order, which precedes μ?
c. How many permutations lie between π *and* μ in the lexicographic order?

3. Prove that the permutations $(1, 2)$ *and* $(1, 2, 3)$ generate S_3, That is every 3-permutation is a product of these permutation (each can be repeated more than once in the product).

4. * The fourty students in our class are standing in a circle in a random order. The dean ordered to rearrange the order of students in the circle so that they will be ordered alphabetically. The only change permitted is for any four consecutive students to reverse their order. So if for example, the students $\ldots A\ B\ C\ D \ldots$ are consecutive on the circle they can be reversed to $\ldots D\ C\ B\ A \ldots$.
Prove that the dean's order can be executed.