

# Discrete Mathematics Drill

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There are a lot more questions that you could answer in one hour. Treat this drill as a review and help each other solve problems in which you have difficulties.

## 1 Permutations

1. What is the location of the permutation  $[8, 1, 4, 3, 9, 11, 2, 5, 7, 6, 10]$  in the Cantor Digits enumeration scheme?
2. Which 10-permutation is in location number 123456?
3. \* A **transposition**  $[i, j]$  is a permutation that exchanges  $i$  and  $j$ . Prove that any product of an odd number of transpositions cannot be the identity permutation.
4. When using the arrow algorithm, in step 1 we get the 9-permutation  $[1, 2, 3, 4, 5, 6, 7, 9, 8]$ . Which permutation will be reached by the arrow algorithm in step 20? step 100?
5. Can you calculate at which step will the 9-permutation  $[1, 3, 5, 7, 9, 8, 6, 4, 2]$  be reached?

## 2 combinations

1. In the lexicographic generation all 5-subsets of  $\{1, 2, \dots, 10\}$  which subset succeeds  $\{10, 4, 7, 9\}$  which subset precedes it?
2. You have five tiles and three different colors. In how many ways can you color the set of tiles?

3. The number of distinct triples from a set  $A$  with  $n$  objects is 35. How large is  $A$ ?
4. The number of distinct subsets of size 7 of the set  $B$  is the same as the number of subsets of size 10. How large is  $B$ ?
5. 45 books (all different) are to be placed on 5 shelves. In how many ways can it be done? (the shelves are distinguishable).
6. 99 Bottles of beer are to be distributed among 5 different tables. In how many ways can it be done?

### 3 binomials

1. Prove that  $\binom{n+1}{x+y+1} = \sum_{k=0}^n \binom{k}{x} \binom{n-k}{y}$ .
2. Let  $S$  be a set with cardinality  $n$ . Prove that  $\sum_{A \subset S} |A| = n \cdot 2^{n-1}$ .
3. Prove:  $\sum_{m=k}^r \binom{m}{k} = \binom{r+1}{k+1}$