

Discrete Optimization

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Name:

1 Assignment-3

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Please submit your answer in a neat, readable properly organized format.

In general, a * in exercises indicates a more challenging problem, ** a highly challenging problem.

1. An oil company plans to explore a new field. They need to rent drill rigs for the exploration. During the exploration, the geologists plan to use different number of rigs during the five months of planned exploration according to the following table:

- First month: 14 rigs.
- Second month: 8 rigs.
- Third month: 12 rigs.
- Fourth month: 5 rigs.
- Fifth month: 20 rigs.

The charges for the rigs are as follows:

- Delivery and setup: \$10000
- Dismantling and return: \$15000
- Rental per month: \$20000.

What strategy should the company use to minimize their expenses?

Try to construct an appropriate digraph so that the shortest path algorithm will be able to identify the cheapest strategy.

2. Prove that Prim's algorithm produces a minimum cost spanning tree.
3. Suppose we have a digraph with edges of negative weight. Suppose we add the same fixed amount to every edge in the graph to make all weights non-negative. Is the shortest path found by Dijkstra's algorithm in the adjusted graph will also find the shortest path in the original graph?
4. Find the shortest path between vertices number 0 and 22 in the graph in the file floyd.pdf.
5. If G is a bipartite graph with partitions P_1, P_2 such that $|P_1| = |P_2|$ and that $\forall A \subset P_1, |N(A)| \geq |A|$ ($N(A) = \{v \in P_2 \mid v \text{ has a neighbor in } P_1\}$). then G has a perfect matching.
6. Do the sets $\{2, 4, 7\}, \{1, 7, 3\}, \{2, 4, 7, 3\}, \{3, 5, 1\}, \{3, 2, 4\}, \{1, 2, 3, 4\}, \{5, 2, 4, 7\}$ have an SDR (set of distinct representatives)?
7. Prove that the seven lines of the Fano plane regarded as triples of points have three SDRs such that every point x represents three different lines in the SDR's.