Tilings and Patterns. By Branko Grünbaum and G. C. Shephard. W. H. Freeman & Co., New York, 1986. 700 + ix pages, 1406 illustrations, cloth, $59.95.

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This is a marvelous book. I am tempted to say "I wish I had written it," but the comprehensiveness and attention to detail exceed anything I would have ever contemplated, let alone executed. The book is profusely illustrated, with an average of two figures per page, and each picture is meticulously drawn. It must have been a difficult decision for the publisher to confine all illustrations to black-and-white, thereby foregoing the art gift book market, though the production cost would no doubt have been exorbitant. However, for the mathematician, all the necessary distinctions are amply illustrated with shadings, letters, and numbers, and an excellent aesthetic effect is achieved.

One major achievement is the organization of the vast literature on planar tilings into a systematic structure, arranged into twelve chapters. The first eight chapters develop a "classical" theory of tilings, including classification of tilings with transitivity properties (Chapter 6), with respect to symmetries (Chapter 7), and when color is added (Chapter 8). Interesting recreational problems are liberally interspersed: rectangles of unequal squares, including Duijvestijn's decomposition of a square into 21 unequal squares (in section 2.4); superposition of two polygonal tilings to obtain a dissection of one polygon where the pieces can be reassembled to form the other polygon, in the fashion of H. Lindgren's 1972 Dover book on *Geometric Dissections* (in section 2.6); and many others.

The last four chapters contain some advanced material, mostly rather recent, and (to this reviewer, at least) especially interesting. Chapter 9, "Tilings by Polygons," includes a wealth of tiling material involving polyominoes and their cousins, polyiamonds and polyhexes, as well as a discussion of spiral tilings such as Voderberg's. Chapter 10, "Aperiodic Tilings," begins with "similarity tilings," including certain *rep-tiles*, which *can* tile aperiodically, and continues with the six-piece sets of Robinson, of Ammann, and of Penrose, which can *only* tile aperiodically. This topic is developed extensively, leading to the two-piece sets of Penrose (the kite-and-dart, and the Penrose rhombs) and Ammann's set (of two similar concave hexagons), which can tile the plane, but only aperiodically. The central unsolved problem is suitably highlighted: *Is there a single tile which can tile the plane aperiodically, but not periodically?* Chapter 11 ("Wang Tiles") develops Hao Wang's theory of the close relationship between Turing machines, undecidability, and the question of whether a given set of colored tiles (actually, "MacMahon squares") can tile the plane. The final chapter ("Tilings with Unusual Kinds of Tiles") discusses tiles with unconventional connectivity (e.g., with reentrant boundaries, and disconnected tiles), tilings by irregularly shaped figures, and tilings with special adjacency conditions on abutting tiles.

Sets of exercises are distributed liberally throughout the book, and it would be possible to use it as a text in a somewhat unconventional geometry course. The book concludes with 42 pages of references, citing articles in sixteen languages, and a six-page index.

The authors have made a commendable effort to cite relevant references for each topic they discuss. My only complaint is that sometimes the *originator* of an idea is not directly credited, if the concept appears in the work of another author who in turn cites the inventor. Thus, Ammann is not listed as an author, which makes it difficult to attribute proper credit to his discoveries. Several of my own contributions (coining the term *rep-tile* and observing the aperiodic tilings they generate; the problem of tiling the plane using exactly one square of each integer side; etc.) are not directly attributed, as the reference given is to *Scientific American* columns by Martin Gardner, or to anthologies of such columns. (This may be mere petulance on my part. I *am* credited with inventing the term *polyominoes* in 1953, and with writing a principal reference on this subject in 1965.)

As extensive as this book is, it obviously cannot present every result ever obtained about tiling. However, the authors have excellent taste in selecting their material, and make an admirable effort to cite references to the topics which they do not cover. In a first edition of a book of this length there are inevitably some typographical errors, but they are surprisingly few and far between. I recommend this book enthusiastically to anyone interested in problems of tiling the plane.