

CHAPTER 5. PROPERTIES OF CONFIGURATIONS.

5.0 OVERVIEW

This chapter differs in character from the preceding ones. While each of them deals with particular kinds of configurations — most concerning their existence and constructions — here we consider certain properties of configurations that have attracted the interest of researchers at various times.

We begin by presenting in Section 5.1 the information available concerning the connectivity properties of configurations. While some of the first papers ignored the issue, it quickly became apparent that restricting the discussion to connected configurations leads to considerable simplification in formulation of results. However, the importance of the degree of connectedness has been slow to emerge. Even today, it is not clear to what extent various properties depend on whether the configuration is 2-connected or has higher degree of connectivity.

Section 5.2 deals with Hamiltonian multilaterals — the analogs of Hamiltonian circuits in graphs. The recent result that there exist 3-connected 3-configurations with Hamiltonian multilaterals is presented; this solve negatively a long standing conjecture that 3-connectedness is sufficient for Hamiltonicity.

The next section deals with the related concept of multilateral decomposition of a configuration. By this is meant a family of multilaterals that include all lines and all points of the configuration, each just once. The topic goes back to the “prehistory” of configurations, and leads to many still open questions.

Section 5.4 is devoted to the presentation of the known facts about configurations that have no trilaterals, or, more generally, no k -laterals for $k = 3, 4, \dots, h$. This is an old topic that has recently been revived, and has turned out to be related to interesting question about graphs.

In Section 5.5 we consider the configurations for which every points is incident with the same number of trilaterals – another old topic in configurations theory.

Section 5.6 is concerned with the recently proposed question what is the largest dimension that a configuration isomorphic to a given one can span. The few results are supplemented by many open questions.

Section 5.7 deals with a topic that has attracted attention only recently — configurations that can be continuously modified while keeping their combinatorial structure and keeping a sizable part of the configuration unchanged. The latest available results are presented, but there is a wide array of open questions to occupy geometers in the future.

Duality and selfduality, and the special cases of polarity and selfpolarity are considered in Section 5.8. The material available shows only too clearly how much is still to be discovered.

The final section, Section 5.9, is devoted to a list of open problems. These are only a few specific questions, meant to illustrate the topics considered. By no means is the list supposed to be exhaustive.