

## 4.8 OPEN PROBLEMS

There is so little known about the various kinds of configurations described in this section that it seems presumptuous to propose specific problems about any of them. But let us try to present a few that would seem capable of being solved within our lifetime.

1. Are any cyclic 5-configurations geometrically realizable? Any cyclic  $k$ -configurations for  $k \geq 6$  ?
2. Develop a theory of  $k$ -astral 5-configurations for some  $k \geq 4$ .
3. Determine whether there exist  $k$ -configurations  $(n_k)$  for all sufficiently large  $n$ , that is for  $n \geq N(k)$ , where  $N(k)$  depends on  $k$  only. Similar question for unbalanced configurations, taking into account the divisibility properties resulting from the symmetry of the incidence relation.
4. Clarify the relation between the configurations  $((4r)_3, (3r)_4)$  for  $r \geq 5$  and cubic curves in the real plane. Can such curves contain all vertices of configurations of this kind for all  $r$  ? Are all such configurations realizable with all vertices on suitable cubic curves? If not, what are the smallest ones that are not realizable in that manner?
5. Consider geometric configurations of points and lines realized in 3-dimensional Euclidean or extended Euclidean space and spanning it. Find some that are astral in that setting, but have no astral realization in the plane.
6. There seems to be no information whatsoever available concerning  $k$ -astral 4-configurations for  $k \geq 3$ .
7. Develop some concept and some results on *configurations of curves* — that is, objects that can be described as "topological configurations of points and circles" in the same sense that configurations of pseudolines are "topological configurations of points and lines".
8. Is it possible to use astral 4-configurations to construct infinite  $k$ -configurations with an accumulation point, for some  $k$  ?