3.9 OPEN PROBLEMS

Among the most intriguing open problems concerning 4-configurations are the following.

1. Is there *any* analog for 4-configurations of Steinitz's theorem (Theorem 2.6.1)? This theorem can be interpreted as saying that every connected combinatorial 3-configuration can be represented geometrically in the plane if one incidence is disregarded. How much of a combinatorial 4-configuration can be realized?

2. Can any of the cyclic configurations (n_4) with generating lines {0,1,6,-3} or {0,1,5,-2}, described in Section 3.1, can be geometrically realized (for any $n \geq 18$)? Cany *any* cyclic 4-configurations be geometrically realized?

3. It is clear that there are some 4-configurations that can be geometrically realized in *rational* plane; as an example we may take the configuration $\mathcal{LC}(4)$ introduced in Section 1.1, and other similarly built configurations. Can any astral (or k-astral) configuration that can be geometric realized (or represented) in the Euclidean plane be realized (or represented) in the rational plane? (It is well known that this cannot be done in a k-astral way.)

4. The astral configuration (24_4) in Figure 3.6.2 and the 3-astral configuration (21_4) in Figure 3.7.1 have the property that the underlying combinatorial configurations have groups of automorphisms that act transitively on the flags of the configuration. (A *flag* consists of a line and a point, incident with each other.) The configuration $\mathcal{LC}(4)$ mentioned above has the same property. Do there exist any other 4-configurations with a single orbit of flags (under automorphisms)?