

CHAPTER 3. 4-CONFIGURATIONS.

3.0 OVERVIEW

As explained in Section 3.1, the first publications dealing with 4-configurations appeared before the end of the nineteenth century, but not much development happened till relatively recently.

In Section 3.2 we present the general results concerning the existence of topological and geometric 4-configurations. The difference between the present case and that of 3-configurations is quite striking – there still are gaps in the numbers n for which we know that an (n_4) geometric configuration exists.

The various methods of construction of reasonably sized (n_4) geometric configurations — all less than twenty years old — are detailed in Section 3.3. These constructions are then applied in Section 3.4 to determine the values of n for which it is known that a geometric 4-configuration (n_4) exist. Although the development of new methods has made the construction of visually understandable 4-configurations possible in many cases, for some of the small numbers there still are only unattractive diagrams, or no known configurations at all.

Section 3.5 sets up the framework for the study of the k -astral 4-configurations; these are the configurations with a very high degree of geometric symmetry.

Based on that, in Section 3.6 we present one of the few complete results about 4-configurations — the complete enumeration of the 2-astral 4-configurations. These are configurations in which there are only two orbits of points, and two orbits of lines, under Euclidean symmetries of the configuration. This topic is related to (and depends upon) the investigation of the intersection-points of diagonals in regular polygons, in itself a subject with a classical flavor but with surprising twists in its unfolding.

Section 3.7 is devoted to 3-astral configurations. The presence of three orbits of points and three orbits of lines results in a family of configurations with properties very different from the ones considered in Section 3.6.

Section 3.8 is concerned with the k -astral 4-configurations with $k \geq 4$. There is again a sea-change in properties compared to 2-astral and 3-astral configurations, as well as in our knowledge of the possibilities.

A few problems not mentioned in the earlier sections are presented in Section 3.9.