CHAPTER 2.   3-CONFIGURATIONS.

2.0   OVERVIEW

This is the longest of our chapters, reflecting not only the long history of the topic of 3-configurations to which the overwhelming majority of the early works was devoted, but also the much more recent discovery of the deficiencies in the original works of a century or more ago, and their correction. We also present the recent developments that center on the study of configurations with a high degree of symmetry.

Section 2.1 investigates the existence of combinatorial, topological, and geometric configurations (n_3) for various values of n.

In Sections 2.2 and 2.3 this is made more detailed by describing the efforts to determine the numbers of distinct configurations (n_3) for specific values of n. These investigations started in the first period of the study of configurations more than a century ago, but have been resumed and advanced only in the recent past.

Section 2.4 is devoted to the attempts to construct all combinatorial configurations (n_3) recursively. This goal seemed to have been achieved by V. Martinetti some 125 years ago — but a few years ago his result was shown to be incomplete. The corrected result was obtained in the doctoral thesis of M. Boben!

Sections 2.5 and 2.6 present the result of the 1894 doctoral thesis of E. Steinitz. This is a remarkable work, even though it has a significant blemish in its geometric part. It is interesting that the last part of this work has remained undeciphered ever since Steinitz wrote it — nobody claims to understand what he is claiming! However, it is clear that at least some parts of the claim are not true.

The next three sections represent recent developments. These are investigations of 3-configurations with remarkably large cyclic or dihedral symmetry groups.

Section 2.10 deals with some unexpected aspects of duality and polarity of these configurations.

Finally, Section 2.11 makes explicit a few of the most intriguing problems about 3-configurations.