## Is Napoleon's Theorem *Really* Napoleon's Theorem?

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**Abstract.** A result frequently attributed to Napoleon Bonaparte is the topic of this note; it has an interesting history, and there is a considerable number of papers devoted to it. Several relevant articles appeared in this MONTHLY. Here we present additional information about the history of the result, supplementing and correcting some of the earlier publications.

Describe equilateral triangles (the vertices being either all outward or all inward,) upon the three sides of any given triangle ABC: then the lines which join the centres of gravity of those three equilateral triangles will constitute an equilateral triangle. Required a demonstration.

This is the earliest known published formulation—reproduced verbatim—of a result (to which we shall refer as *the result*) that is the topic of this note; see Figure 1 for an illustration. The result constitutes the whole of a problem posed by Mr. W. Rutherford, of Woodburn. It was published in 1825, among the "New Mathematical Questions" in *The Ladies' Diary* (see [48]). The demonstration requirement was satisfied in the following year's issue of *The Ladies' Diary* (see [9]). There two proofs were given in detail, with the remark that a "similar demonstration will apply when the vertices ... are turned inward"; the proofs were supplied by several people. Their names are listed as Mr. Tho. Burn, Mr. John Walker, Mr. Mason, Messrs. J. Baines, Tho. Hindmarch, and W. S. B. Woolhouse. Moreover, the editor of The Ladies' Diary "with much regret, omitted several of the elegant demonstrations of this curious property, especially the solution and corollaries of Mr. Isaac Brown." There is no indication that Mr. W. Rutherford had provided a solution. Seeing that for some of the other problems in The Ladies' Diary it is stated that the author of the problem supplied a solution, it may be inferred that Mr. W. Rutherford did not provide a solution. Hence it may be further inferred that he did not have a solution. On the other hand, in view of Rutherford's later achievements (as described in [63]) it is possible he did have a proof; it is strange that he is not listed in Taylor [58]. The reader curious about *The Ladies' Diary* may be interested in the historical details given in Perl [44]. Much of the discussion in [44] centers on the fact that men seem to have been the main contributors to The Ladies' Diary.

The appearance of the result in *The Ladies' Diary* was apparently forgotten for many years. The earliest later mention of Rutherford's question appears in Mackay [**38**, p. 107], where he credits "Dr Rutherford" for the result, but adds, *Probably, however, the theorem dates farther back.* This brief mention is remarkable on two counts. First, it seems that Mackay did not actually see Rutherford's question, since he does not mention that (as formulated by Rutherford) it was a question proposed for solution, and that it did not become a theorem till proved by several people in the following year's issue. Second, even granting that absence of proof is not proof of absence, it is

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496

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Figure 1. An illustration taken from the first published proof of the result; reproduced from *The Ladies' Diary* for 1826, page 38.

hard to agree to the "Probably" part of this quotation. During the almost two centuries since Rutherford's question, no such previous publication was discovered.

Several later writers wondered whether the result may have been known earlier. Among them are Cavallaro [13] and Davis [16]. The former mentions that Torricelli, Cavalieri and Viviani studied the figure formed by attaching equilateral triangles to the sides of an arbitrary triangle, so it is not hard to imagine that one of the three famous mathematicians could have hit upon the result—but Cavallaro admits that there is no trace of it in their published works. Cavallaro fails to note that the result does not appear in his own paper Cavallaro [11] which deals in detail with topics related to the Torricelli points—showing that the result is easy to miss even when discussing related topics. Davis [16] mentions Torricelli and Fermat who "knew about these matters." Going beyond this, Davis states (p. 37) "I conjectured that the whole question was known in antiquity." As Davis is quite a raconteur, it is hard to judge how seriously one should take that statement.

On the other hand, the result was independently discovered by several people, relatively soon after Rutherford. Gerber [26] and Davis [16] have W. Fischer [22] in 1863 as the first discoverer of the result. Davis [16, p. 36] has a facsimile of the first page of W. Fischer [22]. J. Fischer [21], Schmidt [49], and Martini [39] report that the first mention of the result is in a text by G. Turner [60]; I have not seen that text. Chapter 1 of Davis [16] is a story of the author's personal involvement with the result and with Napoleon's theorem. Davis states: "This chapter is a much-expanded version of a talk I gave in 1981 and wrote up shortly after." An *Epilogue* to Chapter 1 of [16] mentions the paper Wetzel [62], which made Davis gives free rein to his imagination and presents an amusing, tongue-in-cheek explanation for the naming of "Napoleon's theorem."

According to Wetzel [62], [*the result*] *is surely one of the most-often rediscovered results in mathematics*. Holmes [31] states the result in 1874 as a fact without reference, and uses it in the proof of another result. Laisant [36, p. 148] mentions in 1877 the result as "propriété bien connue" [of triangles], without finding it necessary to give any specific reference. Cavallaro [13] list also several other sources of the result; most of these are not mentioned in other papers or in the Jahrbuch für die Fortschritte der Mathematik (and were not accessible to me). Closer to current times, additional (pre-

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grunbaum.tex

sumably independent) discoveries continue to be made; for example, Yaglom [64]. It may be of some interest to note that although the result appears in Altshiller-Court's book [2] published in 1925, it was one of the few results omitted from the second edition [3] in 1952.

A companion to the result that has been noted by many (but not all) writers deals with equilateral triangles constructed inwards over the sides of the given triangle (so as to overlap with its interior). The centers of these triangles form another equilateral triangle. There are several interesting relations between the two triangles; several authors have investigated these relations.

In many recent publications the result is called "Napoleon's theorem." Several writers expressed some degree of disbelief in the justifiability of this attribution, but most did not dare to acknowledge explicitly that there is no basis whatsoever for this eponymy. The following facts are relevant to the assignation of Napoleon's name to the result.

It is known and documented that Napoleon Bonaparte maintained reasonably close relations with many of the famous mathematicians of his time. In particular, in 1797 he engaged several of them in discussions concerning Mascheroni's results on constructibility of geometric objects using compasses only. **Napoleon died in 1821, four years before Rutherford published** *the result*. Neither Rutherford, nor any of the solvers of his problem, mentions Napoleon. In fact, during the ninety years following his death, no mention of Napoleon in connection with the result has come to light.

This changed in 1911, with the publication of the 17th edition of Aureliano Faifofer's text [20]. He is quoted (among others by Wetzel [62], Martini [39]) as commenting parenthetically on page 186 about the result (presented as exercise number 494): "Teorema proposto per la dimostrazione da Napoleone a Lagrange." This is the earliest known published mention of Napoleon's name in connection with the result. I checked the accuracy of the quote, except that I do not know for sure that it is not mentioned in any prior edition. Davis [16, p. 54] asserts that "An earlier edition of Faifofer's book contains neither the comment nor the problem." Fritsch [24], J. Fischer [21], and Schmidt [49] mention the 18th edition (in 1912) as the first with Napoleon's name, while Cavallaro [13] and Scriba [52] give the 20th edition (in 1917) as the original source; clearly, these statements are mistaken.

Concerning Faifofer's claim itself, it seems appropriate to invoke the observation made by Christopher Hitchens in another context (see Shermer [55]),

## What can be asserted without evidence can also be dismissed without evidence.

No shred of evidence for Faifofer's statement has come to light in the century since it was made; this leads me (following Schmidt [49]) to the conclusion that it was made without any factual support, for reasons that are unknown. Nevertheless, a strange phenomenon occurred, with countless writers claiming that the result is "attributed" to Napoleon. This is going way beyond Stiegler's "Law of Eponymy" proposed by Stiegler [56], "No scientific discovery is named after its original discoverer." To bolster the claim of validity of the name he bestowed to it, Stiegler notes that the "law" was formulated much earlier, by Robert K. Merton [40]. Stigler also notes many other instances of the law's validity. Napoleon died in 1821 before Rutherford's publication of the result in 1825; as far as two centuries of inquiry indicate, Napoleon seems to be totally innocent of any connection to the result. Thus, not only is the result not named after the original discoverer, it is named after somebody who had nothing to do with it!

As far as I know (after a rather thorough search of the literature), most later mentions of Napoleon's theorem go—directly or indirectly, whether acknowledged or

June–July 2012]

NAPOLEON'S THEOREM?

498

not—back to Faifofer. An exception *appears* to be Campedelli–Gobesso [10]. On pp. 112–113, Campedelli claims without any evidence that the result was presented for proof by Napoleon to the "Italian mathematician" Lagrange. However, I believe that Campedelli got his information from one of the later editions of Faifofer [20]; indeed, his formulation coincides with that of Faifofer (including the unnecessary introduction of the circumcircles of the equilateral triangles attached to the sides of the given triangle), and Campedelli alerted Cavallaro [13] to Faifofer's book in its 20th edition, which appeared in 1917. Cavallaro [12] mentions the result as known, and adds in a footnote that it has been attributed to Napoleon—without a reference; Coxeter [14, p. 23] and the Zentralblatt review (#0020.05007) of Cavallaro [12] mention that it ascribes the result to Napoleon. Coxeter & Greitzer [15] relate that the theorem "has been attributed to Napoleon, though the possibility of his knowing enough geometry for this feat is ...questionable...." This assessment, and its even harsher sounding translation in the German edition of the Coxeter & Greitzer book, is strongly criticized by Schmidt [49] as misinterpreting a statement made by Laplace in 1797.

Hence, by the late 1960's, there have been several mentions of Napoleon's theorem in the literature. The spread of the name continued with Honsberger [**32**, Ch. 3], Nelson [**41**], followed by many others, including myself (Grünbaum [**30**], Shephard [**54**], Alsina & Nelson [**1**]). Many of these and other papers discuss variants and generalizations of Napoleon's theorem; see, for example, Bini [**6**], Rigby [**47**], Wells [**61**], Wetzel [**62**], Schuster [**51**], Bogomolny [**7**], and the survey by Martini [**39**] already mentioned. Many Internet sites present Napoleon's theorem in more or less detail. However, in some of them one can find statements that are not only wrong, but fabricated of whole cloth. One such example is MathPages [**37**], where we read:

On the other hand, since Rutherford's article first appeared in 1825, just four years after Bonaparte died on St Helena, it's also conceivable that Rutherford just decided to name his theorem after the famous fallen Emperor.

As we have seen, Rutherford did not write an "article", or name his problem—not theorem—after anybody, least of all Napoleon.

What conclusions can we draw concerning "Napoleon's theorem"?

First, the name is too widely accepted and used for any revision to have a chance of success. In reality, the name is quite practical. When "Napoleon's theorem" is mentioned, any reader either knows instantly what is meant, or if not—can out find by querying Google or other sources of information. The query "Napoleon's theorem" on Google produced 7,440 results on April 12, 2011. Any one of the first ten gave immediately a usable description. Contrast this with the situation that would occur in similar circumstances if Faifofer had named the result after Gauss or Euler—each of which could certainly have proved the result had it occurred to him. But following similarly a hint to *Gauss' theorem* (81,700 results on Google) or *Euler's theorem* (84,600 results) would take a long time before clarity would be achieved.

Second, and much more important is the fact that the result or Napoleon's theorem is the root from which many other results in elementary geometry developed, as have several very active branches of modern mathematics. Many of the relevant publications are using descriptions such as "Napoleonsétze," "Generalization of Napoleon's theorem" (Kiss [34], Bogomolny [7]), and other similar ones.

Third, in elementary geometry there are many analogues, generalizations or variants that include "Napoleon" in their title or name. Two *Napoleon points* are mentioned by Gale [25] and Kimberling [33], and investigated by Evans [19], among others. *Napoleon's quasigroups* are studied by Krčadinac [35]. *Napoleon configuration* is a

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term used by Martini [**39**] and Boutte [**8**], while the latter considers also two Napoleon *circles*. Martini gives many references to other variants and generalizations of the result.

Less elementary than the above is the "Theorem of Napoleon-Barlotti," first established by Barlotti [4], [5]. It can be stated as follows: The centers of regular *n*-gons constructed over the sides of an *n*-gon *P* form a regular *n*-gon if and only if *P* is an affine image of a regular *n*-gon. Since all triangles are affinely regular, this is a direct generalization of the result. For n = 4 this has been proved earlier by Thébault [59]. The Napoleon-Barlotti theorem was rediscovered by Szabó [57]. Somewhat related are the results of Bini [6].

Next, other generalizations of the result to arbitrary polygons are often associated with Napoleon's theorem. One such generalization that deals with *n*-gons consists of applying n - 2 transformations, each akin to the construction in the Napoleon-Barlotti theorem, but using all-but-one of the n - 1 different regular *n*-gons  $\{n/d\}$ , for  $1 \le d \le n - 1$ . In dealing with oriented *n*-gons, the polygon  $\{n/d\}$  (inscribed in a circle) results from a starting vertex by placing a vertex *j*, for  $0 \le j < n$ , so as to enclose a central angle of  $2\pi j d/n$  to the starting vertex. If *n* and *d* are not coprime, several vertices are represented by the same point. For details see, for example, Grünbaum [27], [28]. The end-result is a regular *n*-gon of the one kind that has not been used in the construction. This was first established by Petr [45], [46], and later independently by Douglas [17], [18] and Neumann [42], [43]. Several other authors discuss variants and generalizations of this result.

Another family of results developed from studies of variants of the result, and frequently are considered as relatives of Napoleon's theorem. They deal with iterations of the basic construction—the attachment of regular polygons to sides of a given or previously obtained polygon—and with the limiting behavior of such processes. See, for example, Schuster [50], Grünbaum [29], Shephard [53].

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June–July 2012]

NAPOLEON'S THEOREM?

499

page 500

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500

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