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STILL MORE RHOMBIC HEXECONTAHEDRA

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In a paper [1] published last year in GEOMBINATORICS I conjectured that the only rhombic hexecontahedra (that is, polyhedra having as faces sixty congruent rhombi, which are equivalent under symmetries of the polyhedron) are the two shown in Figures 1 and 2. The former was discovered by Unkelbach [3] more than sixty years ago, while the latter was first presented in [1]. (Comments regarding this polyhedron can be found in [2].) In the intervening period I came to realize that this conjecture is not just wrong, but grievously wrong. This may sound somewhat oxymoronic — but the truth is that there is not just one counterexample, but quite a few — close to a dozen. It is hard to pin down the precise number, since it depends on somewhat arcane details of the definition of "polyhedron". Pursuing this would lead us far beyond the framework of this article; the more complete enumeration will be published elsewhere. However, two of the counterexamples to my conjecture are relatively easy to describe; they are shown in Figures 3 and 4, and their presentation does not require

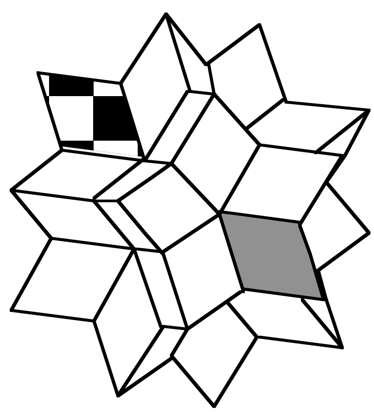

Figure 1. The rhombic hexecontahedron first described by Unkelbach [3] in 1940. It is free of selfintersections, and each of its faces is coplanar with one other face. The corners of each face are on one 3-valent convex vertex, one 5-valent concave, and two 4-valent saddle-shaped vertices of the polyhedron.

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delving deeply into what is and what is not a polyhedron. The new examples are no worse, as polyhedra with selfintersections go, than the earlier example shown in Figure 2, or the well-known Kepler-Poinsot regular polyhedra. Their descriptions are best understood on hand of the diagrams, whose captions contain additional information.

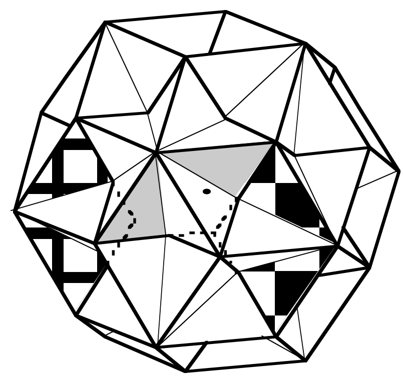

Figure 2. The rhombic hexecontahedron first described in [1]. Each face crosses two other faces, and is coplanar with one other face. Two corners of each face are on 6-valent nonconvex vertices of the polyhedron, one is on a 3-valent vertex which is "inside" the polyhedron, and one is on a 5-valent pentagram-shaped vertex.

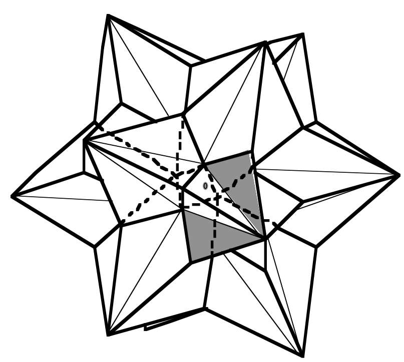

Figure 3. A rhombic hexecontahedron in which each face intersects two other faces, and no two faces are coplanar. The corners of each face lie on two saddle-shaped 4-valent vertices, one 5-valent concave vertex inside the polyhedron, and one 5-valent pentagram-shaped vertex.

Straightforward calculations show that the faces of the polyhedra shown in Figures 3 and 4 are indeed rhombi. However, it is of some interest to note that while the rhombi forming the polyhedra in Figures 1 and 2 are "golden rhombi", having the "golden number" $\tau = (1+\sqrt{5})/2 = 1.618034...$ as ratio of diagonals, the new rhombohedra in Figures 3 and 4 have diagonals in the ratio $\sqrt{2} = 1.414214...$.

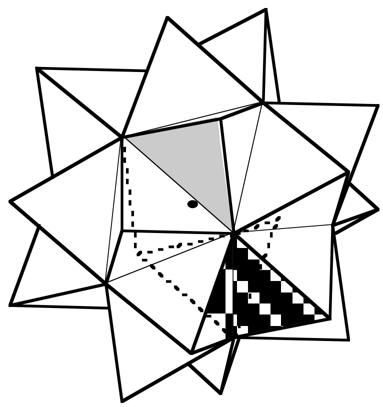

Figure 4. A rhombic hexecontahedron in which each face intersects one other face, and no two faces are coplanar. The corners of each face lie on one 3-valent vertex, one 5-valent convex vertex inside the polyhedron, and two 10-valent star-shaped vertices.

References.

[1] B. Grünbaum, A new rhombic hexecontahedron. Geombinatorics 6(1996), 15 - 18.

[2] B. Grünbaum, A new rhombic hexecontahedron — once more. Geombinatorics 6(1996), 55 - 59.

[3] H. Unkelbach, Die kantensymmetrischen, gleichkantigen Polyeder. Deutsche Mathematik 5(1940), 306 - 316. Reviewed by H. S. M. Coxeter in Math. Reviews 7(1946), p.164.