From: Geombinatorics 6(1996), 55 - 59.

A NEW RHOMBIC HEXECONTAHEDRON — ONCE MORE

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Several people commented on the preprint and published versions of my paper [2] — hence this note to enlarge upon, add to, and clarify the earlier statements.

The rhombic hexecontahedron of Figure 1 of [2], reproduced again in Figure 1 below, has been (implicitly) described in several papers that deal with stellations of the rhombic triacontahedron, and it also was found by other people who apparently did not publish their results. However, I did not feel it necessary to mention the stellation papers in [2], since

(i) I stated the fact that its kernel is the rhombic triacontahedron; since every face-transitive and selfintersection-free polyhedron is a stellation of its kernel, it clearly follows that this hexecontahedron is a stellation of the triacontahedron;

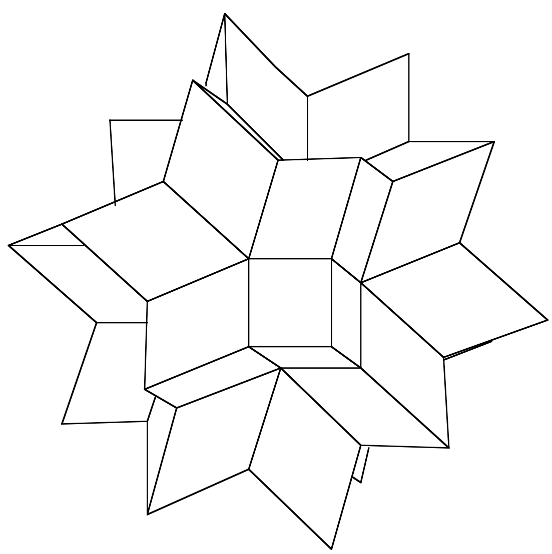

Figure 1. The rhombic hexecontahedron described by Unkelbach [4] in 1940.

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1 Research supported in part by NSF grant DMS-9300657.

(ii) none of the discussions of stellations of the rhombic triacontahedron that were known to me preceded the reference to Holden [H] for this hexecontahedron given in [2] (and the same is true for the references brought to my attention since then);

(iii) the papers on stellations neither provided an illustration of this hexecontahedron, nor singled it out among the considerable number of other stellations they described. (Only in [3] is there an offer to send interested readers a Supplement; this Supplement does contain a diagram of the hexecontahedron in question, as one of a series of "notable, fully supported stellations".)

Some readers seem to have obtained the impression that I had meant to say that the polyhedron in Figure 1 is a "new hexecontahedron". This impression may in part be due to the fact that the editor placed a copy of Figure 1 on the title page of the issue of GEOMBINATORICS in which [2] appears, and provided a caption (on the inside front cover) which reads "The cover illustration comes from the essay *A New Rhombic Hexecontahedron*, ...". However, none of this justifies attributing to me any claim of priority, especially in view of the introductory paragraph of [2] which concludes by stating that this hexecontahedron "... appears in the very attractive and unusual book by Holden [5], published in 1971."

Since [2] was written, I found what almost certainly is the first publication which explicitly mentions the hexecontahedron, and includes even a photo of a model; it is much earlier than any of the other references I have. This is the paper [4][2] by Helmut Unkelbach, which appeared in 1940 ! In his review [1] of [4], Coxeter writes that Unkelbach presents, among other polyhedra, "... a remarkable rhombic hexecontahedron ... illustrated by a photograph of a model. ... Its faces have the same shape as those of the triacontahedron, of which it is actually a stellation."

The "new rhombic hexecontahedron" to which the title and text of [2] refer is the one described and illustrated in [2], as well as here, in Figures 2 and 3. While I believed when writing [2], and believe now, that [2] is the first description of this polyhedron, the attentive

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2 Although the journal in which the paper [4] appeared was started mainly to serve as a propaganda forum for the Nazi philosophy, and several of the articles that appeared in it were so disgusting that a reprint of the journal after World War II provided blank pages instead of them, some other articles contained valuable mathematics. Unkelbach's paper [4] is one of these.

reader will observe that each time the word "new" was used in the text and captions, it was in quotation marks. The reason for this hedging is that although I am not aware of any prior description of this polyhedron, its relatively simple construction may well have been observed in some prior publication. If any reader knows of such a publication, or of an earlier than [4] description of Unkelbach's hexecontahedron, I would appreciate learning about it.

Due to technical difficulties in printing the journal, Figure 2 of [2] came out somewhat mangled. In the text it is indicated that "three faces ... have been highlighted by shading" — meaning, naturally, by **distinct** shades, black and two different gray levels. Unfortunately, the darker of two of the gray shadings came out black, thus complicating the understanding of the structure of the polyhedron. In Figure 2 shown below a different set of distinguishing shadings has been used, in the hope that they will reproduce distinctly. To further ease the visualization of this hexecontahedron, in Figure 3 it is shown as it would appear if the three faces that partially hide the three shaded faces were transparent, so that the shaded faces can be seen in their entirety. The visible edges of the transparent faces have been rendered in white, and this applies also to their intersections with the shaded faces; these intersections are not edges of the polyhedron.

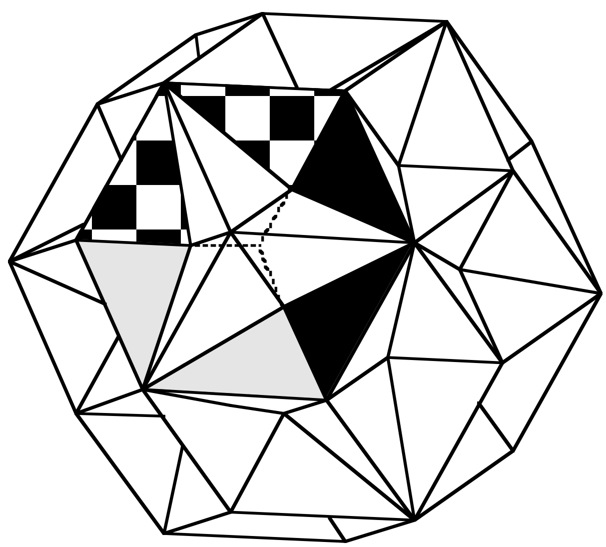

Figure 2. A version of Figure 2 of [2] in which a different shading is used to highlight three of the rhombi.

In visualizing the "new" hexecontahedron it is important to realize that it consists of two "layers" which are interwoven at the 5-valent vertices in the same way that the triangles or pentagons are interwoven at the vertices of the well known Kepler-Poinsot regular polyhedra usually denoted $\{3,5/2\}$ and $\{5,5/2\}$, the *great icosahedron* and the *great dodecahedron*. On the other hand, beneath every 6-valent vertex there is a 3-valent one.

Dr. Peter Messer kindly brought to my attention that among the stellations of the rhombic triacontahedron listed in [3], the one designated as J in Table 3 of [3], and illustrated in the Supplement obtainable from the author, looks exactly like the "new" hexecontahedron in Figures 2 and 3. In fact, although their models may appear to be the same, Messer's stellation J and the rhombic hexecontahedron are completely different polyhedra. By their definition, stellations[3] are **solids**. The **boundary** of the stellation J consists of 120 triangles, which are the **parts** of the 60 rhombi of Figures 2 and 3 that are visible from the outside (each rhombus of the hexecontahedron has two triangular parts visible from the outside, together with a kite-shaped quadrangular part not visible from the outside). In other words, the union of all the bounded 3-dimensional cells determined the rhombic hexecontahedron coincides with the stellation J, but the stellations yields no information about the rhombi which constitute the hexecontahedron, nor does it distinguish between the cells enclosed once and those enclosed twice by the hexecontahedron.

It is my sincere hope that the above explanations will have completely clarified all questions that may have arisen in connection with [2].

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3 For the convenience of readers not familiar with stellations, here is a thumbnail sketch of this concept. Given a polyhedron P, the **planes** of the faces of P partition the 3-dimensional space into a finite number of convex **cells**. Any union of some collection of these cells is (in principle) a stellation of P. In practice, the starting polyhedron P is always convex and very symmetric (regular, Archimedean, isohedral, etc.) and only stellations which have the same, or nearly the same, symmetry group as P are considered. Most authors restrict the usually still huge number of stellations by considering only finite cells, by choosing the cells as contiguous, and by imposing various other conditions deemed appropriate for their studies.

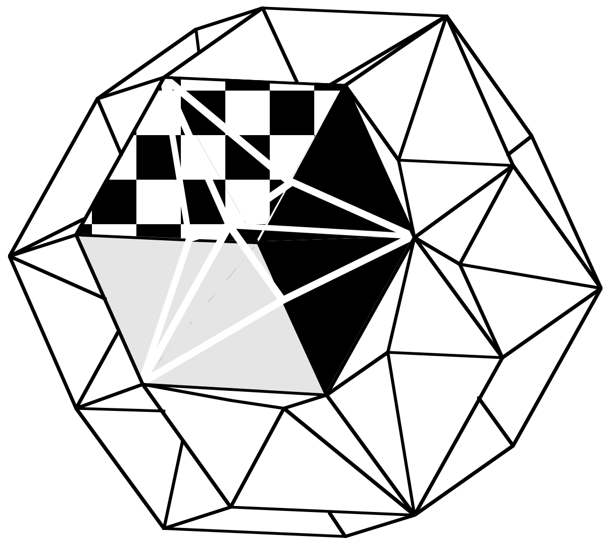

Figure 3. A different view of the "new" rhombic hexecontahedron. In order to show the three shaded faces completely, the three faces that partially obscure them have been rendered as transparent, their edges (and the intersections with the shaded faces) in white.

References.

[1] H. S. M. Coxeter, review of [4]. Math. Reviews 7(1946), p.164.

[2] B. Grünbaum, A new rhombic hexecontahedron. Geombinatorics 6(1996), 15 - 18.

[3] P. Messer, Stellations of the rhombic triacontahedron and beyond. Structural Topology 21(1995), 25 - 46. A "Supplement" with illustrations of many of the stellations is available from the author on request.

[4] H. Unkelbach, Die kantensymmetrischen, gleichkantigen Polyeder. Deutsche Mathematik 5(1940), 306 - 316.