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Patterns on the 2-sphere.

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A pattern on the ordinary sphere is a set of disjoint congruent copies M_i of a motif M (usually a closed topological disk or simply a dot) such that the symmetry group S of the pattern is transitive on the M_i. For the purpose of classification into a finite number of types, two patterns, P and P', are said to be henomeric if, when suitably superposed, they have the same symmetry group acting the same way on both patterns. In other words, the copies M_i and M_i' in the two patterns can be put into one-to-one correspondence so that each generator of S is represented by the same permutation of the subscripts i in both patterns. In a slightly more refined classification, P and P' are said to be homeomeric if they are henomeric and the correspondence can be effected by a continuous transformation. In particular, isogonal polyhedra can be classified by regarding their sets of vertices as dot patterns. For instance, the truncated tetrahedron and nonuniform cuboctahedron (H and I on page 22) can be seen to be henomeric by reversing each triangular face of the former; but they are not homeomeric because a continuous process of reversal would make the triangle collapse to a single point in the middle of the process. More generally, any finite pattern in space can be subjected to this kind of classification by taking its centroid to be the center of a sphere, projecting the pattern centrally onto this sphere, and shrinking the consequent M_i till they become disjoint.

The authors have made perspective drawings of the 34+22 types of discrete closed disk patterns (the 22 being examples taken from 22 families involving an integral parameter q), and of the 24+5 homeomerically distinct isogonal polyhedra. They have also provided extraordinarily detailed tables, many other drawings (such as patterns of great circles on the sphere and of lines in the projective plane), and an impressive list of references. For an analogous (but brief) account of infinite patters in the Euclidean plane, see the authors' earlier note [C. R. Math. Rep. Acad. Sci. Canada **1** (1978/79), no. 1, 57–60; MR0511841 (80c:51014)].

Reviewed by *H. S. M. Coxeter*