The generalized Cauchy equation for operator–valued functions

A. B. Buche and H. L. Vasudeva

Let $\mathfrak{X}$ denote a Banach space and let $\mathscr{B}(\mathfrak{X})$ denote the family of bounded linear operators on $\mathfrak{X}$. Let $\{S(t); t \in R^+\}$, $S: R^+ \to \mathscr{B}(\mathfrak{X})$ be a one-parameter family of operators and let $H: \mathscr{B}(\mathfrak{X}) \times \mathscr{B}(\mathfrak{X}) \to \mathscr{B}(\mathfrak{X})$ be a function. The family $\{S(t)\}$ is said to be a *generalized Cauchy system* if it satisfies the generalized Cauchy equation $S(s+t) = H(S(s), S(t))$, $s, t \in R^+$. Let φ and ψ be real-valued functions defined on R^+ such that (i) $\lim_{t \to 0^+} \varphi(t) = 0$, (ii) φ is a non-negative strictly monotone increasing continuous function on $R^+ - \{0\}$, (iii) ψ is a non-negative function, bounded on each compact set of R^+, (iv) $\|H(S(s_1), S(t)) - H(S(s_2), S(t))\| \leqslant \varphi(\|S(s_1) - S(s_2)\|)$. $\psi(\|S(t)\|)$, $s_1, s_2, t \in R^+$. Then, in the uniform operator topology, the Lebesgue measurability of S on $R^+ - \{0\}$ implies continuity of S on $R^+ - \{0\}$.

Received February 11, 1974.

Pairs of edge-disjoint Hamiltonian circuits

Branko Grünbaum and Joseph Malkevitch

Several authors have conjectured that there exists a pair of edge-disjoint Hamiltonian circuits in every 4-valent 4-connected graph, or at least in each planar graph of this kind. A negative answer to the general conjecture follows from a recent result of G. H. J. Meredith; the planar variant was recently refuted by Pierre Martin. We provide simpler counterexamples to both versions of the conjecture. One of our methods may also be used to establish that every 3-connected, 3-valent, cyclically-4-connected planar graph has a simple circuit that contains at least $\frac{3}{4}$ of the vertices, while for some such graphs (with arbitrarily many vertices) no simple circuit contains more than $\frac{76}{77}$ of the vertices.

Received March 25, 1974.

Matrix summability and a generalized Gibbs phenomenon

J. A. Fridy

Consider a real-valued function sequence $f = \{f_k\}$ that converges to φ on a deleted neighborhood of α. If there is a subsequence $\{f_{k(j)}\}$ and a number sequence x such that $\lim_j x_j = \alpha$ and either

$$\lim_j f_{k(j)}(x_j) > \limsup_{x \to \alpha} \varphi(x) \quad \text{or} \quad \lim_j f_{k(j)}(x_j) < \liminf_{x \to \alpha} \varphi(x),$$