Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 9.3—Representing Graphs and Graph Isomorphism

Extra
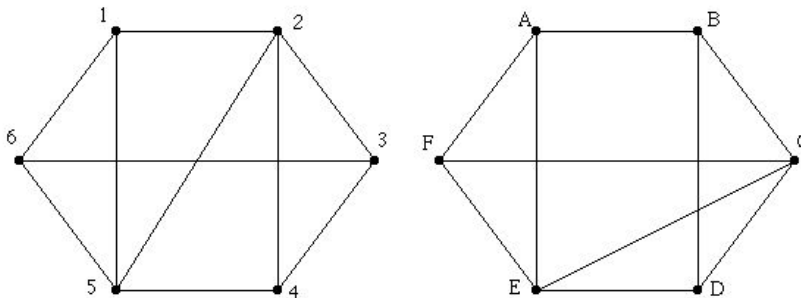**Examples** — *Page references correspond to locations of Extra Examples icons in the textbook.*

#1. Determine whether the following graphs are isomorphic.
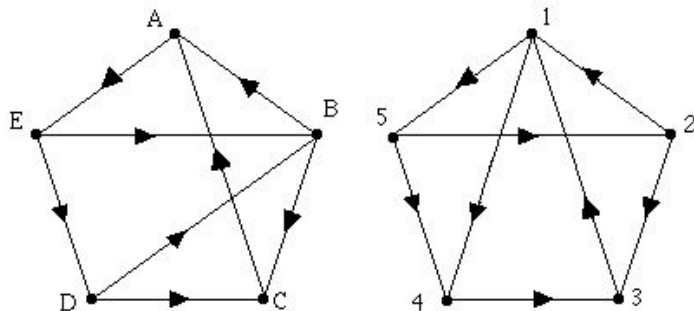

Solution:

The graphs are isomorphic. In the graph on the left, only vertices 2 and 5 have degree four. In the graph on the right, only vertices C and E have degree four. Therefore, if the two graphs are to be isomorphic, , we must have 2 and 5 correspond to C and E as either 2-C, 5-E, or as 2-E, 5-C. Either correspondence gives rise to an isomorphism:

$$1\text{-}F, \ 2\text{-}C, \ 3\text{-}B, \ 4\text{-}D, \ 5\text{-}E, \ 6\text{-}A.$$

$$1\text{-}D, \ 2\text{-}E, \ 3\text{-}A, \ 4\text{-}F, \ 5\text{-}C, \ 6\text{-}B.$$

#2. Determine whether the following digraphs are isomorphic.

Solution:

Even though the graphs have many features in common (such as the same number of vertices, the same number of edges, matching in-degrees and out-degrees), the digraphs are not isomorphic.

Here is one reason: Vertex B must correspond to vertex 1 because they are the only vertices with in-degree 2

and out-degree 2. Vertices D and E each have in-degree 1 and out-degree 2. If the two graphs are to be isomorphic, then D and E must correspond to 2 and 5 (in some order). Because there is an edge from E to D, there must be a corresponding edge in the digraph on the right — this forces D to correspond to 2 and E to correspond to 5. However in the left graph there is an edge from E to B, but no edge from 5 to 1 (the vertices corresponding to B and E) in the right graph. Therefore, the two digraphs are not isomorphic.