Discrete Mathematics and Applications

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Hanoi 2012
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1 Assignment No. 8: Counting

Due: Tuesday, Nov. (to be announced)

Please submit your answer in a neat, readable properly organized format.

2 Elemntary counting

1. A multiple choice test has 40 questions. Each question has four possible answer (with only one correct answer).
   a. In how many ways can a student answer all 40 questions?
   b. In how many ways can a student answer all question so that exactly 30 questions are answered correctly?
   c. In how many ways can a student answer the questions if he can leave some answers blank?
2. How many positive integers < 10000:
   a. Are divisible by 13.
   b. Are divisible by 13 or by 15.
   c. Are divisible by 13 and by 17 but not by both.
   d. Are relatively prime to 13 and 17.
3. A **palindrome** is a string whose reversal is the same as the original string, ABBA is a palindrome. How many bit strings of length n are plaindromes?

2.1 Proofs

1. Prove that among any $n + 1$ positive integers none greater than $2n$ there must be a pair of integers n, m such that n divides m (notation: $n \mid m$).

2. a. Prove that there is an integer $k < 1000$ such that the absolute value of the difference between $k\sqrt{2}$ and its nearest integer is $< \frac{1}{1000}$.

   b. * Let x be an irrational number and n a positive integer. Prove that there is an integer $j \leq n$ such that the absolute value of the difference between jx and the nearest integer to jx is less than $\frac{1}{n}$.

3. Prove that there are infinitely many prime numbers of the form $4k + 3$.

4. (**SAGE**): 11 is a palidnrome which is also a prime. Can you find a prime p which is a palindrome and is 10 digits long?

5. (**SAGE**) Is there a prime number among all integers with 10 digits that contains each decimal digit exactly once?