# Discrete Mathematics and Applications

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## 1 Assignment No. 5: mathematical induction and finite sets

Due: Tuesday, 23 Oct.

Please submit your answer in a neat, readable properly organized format.

This assignment includes a section on mathematical induction. We hope that we do not need to review it in class, it is just a refresher and preparation for some future topics.

1. Construct the largest family of subsets of $\{1, 2, \ldots, 8\}$ such that every two subsets have exactly two members in common.
2. Are the incident vetors of your sets linearly independent?
3. Construct a family of triples, each a subset of $\{1, 2, 3, \ldots, 9\}$ such that every pair $\{i, j\}$ appears in exactly one triple.
4. The city council of Oz has 13 members. The mayor of the city wanted to distribute responsibilities among the council members. He ordered them to form 13 committees. Each committee is to have 4 members. Each council membercan serve on no more than four committees. Can you help design the committees?

## 2 Mathematical Induction

1. Prove that if $k$ is odd then $2^{n+2}$ *divides* $k^{2^n} - 1 \quad \forall n \in Z^+$.
2. a. Prove that $(1+\sqrt{2})^{2n} + (1-\sqrt{2})^{2n}$ is an even integer.
   b. Prove that $(1+\sqrt{2})^{2n} - (1-\sqrt{2})^{2n} = k\sqrt{2}$ for some positive integer $k$.
3. Prove that $\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1$

4. Prove that $\sqrt{n} \leq \sum_{k=1}^{n} \frac{1}{\sqrt{k}} \leq 2\sqrt{n}$

5. Recall the Fibonacci sequence:

   $1, 1, 2, 3, 5, \ldots$ or $f_1 = f_2 = 1,\ f_i = f_{i-1} + f_{i-2}\ for\ i \geq 3$

   Prove that $f_k f_n + f_{k+1} f_{n+1} = f_{n+k+1} \quad \forall n, k \in Z^+$.

6. * Prove by induction the *means inequality:*

$$\frac{1}{n}(\sum_{i=1}^{n} a_i) \geq (a_1 a_2 \ldots a_n)^{\frac{1}{n}}$$