# Discrete Mathematics <br> Drill-3 

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## 1 Drill

## 1.1 sets

1. True or False: if $P(A)=P(B)$ then $A=B$ ?
2. True or False: if $B \oplus A=C \oplus A$ then $B=C$ ?
3. Use unions, intersections and complements to describe the set: $(A \oplus B) \cup \overline{(B \oplus C)} \cap(B \oplus C)$

### 1.2 Functions

All functions are below are defined on the set $N_{75}=\{0,1, \ldots, 74\}$. Which function is a bijection?

1. $f(n)=(5 n-23) \bmod 75$
2. $f(n)=13 n+25 \bmod 75$
3. $f(n)=52 n-25 \bmod 75$
4. $f(n)=n^{2}-3 n \bmod 75$
5. For each of the above functions find $f^{-1}(10)$ if it exists.
6. In the enumeration of $\mathbf{N} \times \mathbf{N}$ discussed in class which pair will be in location 3051 ? What will be the index of the pair $(50,50)$ ?
7.* Let $\mathbf{I}=\{x \mid 0 \leq x \leq 1\}$.

Construct a function $f: \mathbf{I} \rightarrow \mathbf{I}$ such that $\forall y: 0 \leq y \leq 1$ there are exactly two numbers $x_{1}, x_{2} \in \mathbf{I}$ such that $f\left(x_{1}\right)=f\left(x_{2}\right)=y$.
$8^{* *}$ Can you construct a bijection between the unit interval and the unit square?

### 1.3 Sequences

For the following sequences find a rule that governs them and show the next two numbers in the sequences:

1. $2,5,10,17,26, \ldots$
2. $1,3,6,10,15, \ldots$
3.     * $2,2,3,5,5,7,7,11,11,11,11,13,13,17,17,17, \ldots$
