Discrete Mathematics Drill-3

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1 Drill

1.1 sets

- 1. True or False: if P(A) = P(B) then A = B?
- 2. True or False: if $B \oplus A = C \oplus A$ then B = C?
- 3. Use unions, intersections and complements to describe the set: $(A \oplus B) \cup \overline{(B \oplus C)} \cap (B \oplus C)$

1.2 Functions

All functions are below are defined on the set $N_{75} = \{0, 1, ..., 74\}$. Which function is a bijection?

- 1. $f(n) = (5n 23) \mod 75$
- 2. $f(n) = 13n + 25 \mod 75$
- 3. $f(n) = 52n 25 \mod 75$
- 4. $f(n) = n^2 3n \mod 75$
- 5. For each of the above functions find $f^{-1}(10)$ if it exists.
- 6. In the enumeration of $\mathbf{N} \times \mathbf{N}$ discussed in class which pair will be in location 3051? What will be the index of the pair (50, 50)?
- 7.* Let $\mathbf{I} = \{x \mid 0 \le x \le 1\}$. Construct a function $f : \mathbf{I} \to \mathbf{I}$ such that $\forall y : 0 \le y \le 1$ there are exactly two numbers $x_1, x_2 \in \mathbf{I}$ such that $f(x_1) = f(x_2) = y$.
- $8^{\ast\ast}$ Can you construct a bijection between the unit interval and the unit square?

1.3 Sequences

For the following sequences find a rule that governs them and show the next two numbers in the sequences:

- 1. $2, 5, 10, 17, 26, \ldots$
- 2. $1, 3, 6, 10, 15, \ldots$
- $3. * 2, 2, 3, 5, 5, 7, 7, 11, 11, 11, 13, 13, 17, 17, 17, \dots$