Counting

Ngày 16 tháng 11 năm 2012

Question

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To answer these questions we shall start practicing counting using common sense.

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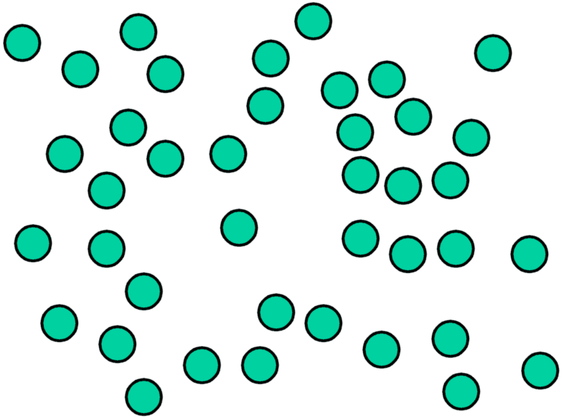
A list of counting problems can be found in the file Let Us Count.pdf.

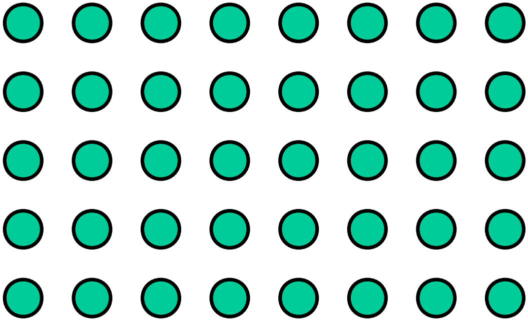
Làm thế nào nhiều trứng được vận chuyển trên các xe gắn máy trong ảnh?

How many students are attending this class?

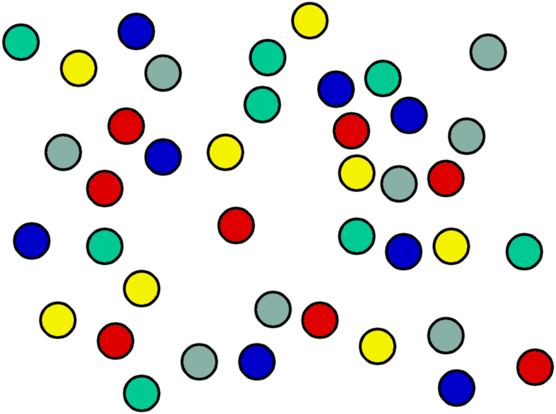
How many green disks are in this picture?



Can you count now?



And how about now?



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2. If a collection can be “organized” (physically or conceptually, for example the “green” rectangular array) it can help us count the number of objects in the collection.
3. If the collection can be partitioned into “smaller” collections, in particular if every smaller collection has the same number of objects, it may again help us count.

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4. Return the largest weight.

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6. Calculate: $\sum_{i=1}^{n-1} j \times (n-j) = \frac{1}{6}(n^3 - n)$

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MORE THAN 27 YEARS!

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