

Discrete Mathematics

Drill-5

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1 Finite sets

1. Construct a family of subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ such that:
 - Every subset has an even size.
 - Every two subsets have an even number of members in common (0 is even).
 - What is the largest number of such subsets you can construct?
2. Construct a family of subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ such that:
 - Every subset has an odd size.
 - Every two subsets have an even number of members in common (0 is even).
 - What is the largest number of such subsets you can construct?
3. Construct a family of subsets of $\{1, 2, 3, 4, 5\}$ such that every two subsets have exactly two numbers in common.
4. How many elements does the set $P(P(P(\emptyset)))$ have?
5. Let $A = \{a, b, c, d, e\}$
 - (a) What is the characteristic (incidence) vector of $\{a, b, e\}$?
 - (b) What is the characteristic vector of $\{a, b, c, d\} \cap \{b, c, d, e\}$?
 - (c) List all characteristic vectors of all subsets of A with cardinality 4.
6. Let $S = \{1, 2, \dots, 7\}$ Find a family of 7 triples such that any pair of distinct triples have exactly one number in common.
7. Prove that the 7 characteristic vectors of the triples you found are linearly independent.

8. *Can you prove that it is not possible to find 8 triples such that every two triples have exactly one number in common.