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2. Symmetric if $a \propto b \to b \propto a$
3. Transitive if $a \propto b \wedge b \propto c \to a \propto c$

# Question 1

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## Remark

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## Remark

*This is an order relation on the positive rational numbers in which every non empty subset has a minimal element.*

# Question 4

Let $m \propto n,\ n, m \in z^+$ if when there is a 1 in the binary representation of m there is also a 1 in the binary representation of n.

## Example

$12 = 1100_2,\ 18 = 10010_2\ \ so\ \ 12 \not\propto 18$
$21 = 10101_2\ \ so\ \ 5 \propto 21$

Prove that this is a partial order on $Z^+$.

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## Proof.

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2. This relation is clearly reflexive.
3. It is antisymmetric. If $n \neq m$ and $n \propto m$ *then* $m \not\propto n$
4. It is transitive! Follows directly from the definition.

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*Let* $n \propto m$ *if* $\frac{a^3+b^3}{ab} \in Z^+$.
*Reflexive:* $m \propto m$ *becuase* $\frac{a^3+a^3}{a^2} = 2a \in Z^+$
*Symmetric:* $m \propto n \rightarrow \frac{m^3+n^3}{nm} \in Z^+ \rightarrow n \propto m$
*Transitive: This relation is not transitive.* $2 \propto 4 \wedge 4 \propto 8$
*but* $2 \not\propto 8$ *becuase* $\frac{2^3+8^3}{16} = 32\frac{1}{2} \notin Z^+$.

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2. Is the transitive closure symmetric?
3. Is the transitive closure reflexive?