Discrete Mathematics and Applications

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Hanoi 2012
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1 Assignment No. 1

Due: Tuesday, Sep. 25

Please submit your answer in a neat, readable properly organized format. A * in an exercise indicates a more challenging problem. A ** is very challenging

2 Logic

1. Construct the truth table of $(p \to q) \vee (\neg p \to r)$
2. Construct the truth table of $(p \to q) \wedge (\neg p \to r)$
3. In a remote village in the mountains all men are either politicians or teachers. Politicains always lie while teachers always tell the truth. You meet two men, Hoang and Tu. Hoang says "we are both teachers". Tu says "Hoang is a politician". Determine who is a teacher and who is a politician.

3 Proofs

1. The Grand Master takes a set of 8 stamps, 4 red and 4 green, known to the logicians, and loosely affixes two to the forehead of each logician so that each logician can see all the other stamps except those 2 in the Grand Master's pocket and the two on her own forehead. He asks them in turn if they know the colors of their own stamps:
   - A: "No."
   - B: "No."
   - C: "No."
   - A: "No."
   - B: "Yes."

- What color stamps the master has in his pocket? What color stamps B has on his forehead?

2. Prove that if a, b are real positive numbers then $a^2 + b^2 \geq 2ab$.

3. Prove that if a, b, c are real positive numbers then $a^3 + b^3 + c^3 \geq 3abc$.

4. * Construct 6 points in general position in the plane R^2 (no three on a line) such that one distance appears 5 times, a second distance appears 4 times, a third distance appears 3 times, a fourth distance appears twice and one distance appears once.

5. Let S be the set of integers of the form $n^2 + 2m^2$. For instance, $9, 22$ are in S : $9 = 1^2 + 2 \times 2^2$, $22 = 2^2 + 2 \times 3^2$. Find five consecutive integers in S and prove that S does not contain six consecutive integers ($7, 8, 9, 10$ are consecutive integers).

4 SAGE exercises

1. Let n be the sum of the digits of your cell phone number. Let m be the number obtained from n by appending the digit 7 at the end. For example, if n 46 then m = 467. Find an integer q such that $mq = 11....1$.

2. $\pi = 3.14...$ Find the first occurence of a prime consisting of 20 consecutive digits in the decimal expansion of π.