Discrete Mathematics
Lecture-15: Number Theory Quick Review

November 17, 2012

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- Returns a^b mod c.

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- **Primitive Roots:** The finite field $GF(q)$ has primitive roots ($\{a^k,\ 0 \leq k \leq q-2\} = GF^*(q)$).