

Discrete Mathematics 2012

Lecture 1

Ngày 25 tháng 8 năm 2012

1. Introduction

Question

What is mathematics?

One of my favorite

1. Introduction

Question

What is mathematics?

One of my favorite

Definition

Mathematics is the study of numbers, shapes and patterns

1. Introduction

Question

What is mathematics?

One of my favorite

Definition

Mathematics is the study of numbers, shapes and patterns

Question

How do you study mathematics?

1. Introduction

Question

What is mathematics?

One of my favorite

Definition

Mathematics is the study of numbers, shapes and patterns

Question

How do you study mathematics?

Answer

With love and enthusiasm.

Question

How do you really study mathematics?

Introduction

Question

How do you really study mathematics?

Answer

Introduction

Question

How do you really study mathematics?

Answer

① *Observe*

Introduction

Question

How do you really study mathematics?

Answer

- 1 *Observe*
- 2 *Hypothesize*

Question

How do you really study mathematics?

Answer

- 1 *Observe*
- 2 *Hypothesize*
- 3 *Verify*

Question

How do you really study mathematics?

Answer

- 1 *Observe*
- 2 *Hypothesize*
- 3 *Verify*
- 4 *Modify (if needed)*

Question

How do you really study mathematics?

Answer

- 1 *Observe*
- 2 *Hypothesize*
- 3 *Verify*
- 4 *Modify (if needed)*
- 5 *Prove*

Question

How do you really study mathematics?

Answer

- 1 *Observe*
- 2 *Hypothesize*
- 3 *Verify*
- 4 *Modify (if needed)*
- 5 *Prove*

Question

How do you really study mathematics?

Answer

- 1 *Observe*
- 2 *Hypothesize*
- 3 *Verify*
- 4 *Modify (if needed)*
- 5 *Prove*

Let us start with a small sample of examples.

Examples

Examples

- ① • Consider the sequence 1, 2, 4,

Examples

- ① • Consider the sequence 1, 2, 4,

Examples

- ① • Consider the sequence 1, 2, 4, 7,

Examples

- ① • Consider the sequence 1, 2, 4, 7, 11,

Examples

- ① • Consider the sequence 1, 2, 4, 7, 11, 16,

Examples

- ① • Consider the sequence 1, 2, 4, 7, 11, 16, 22...

Examples

- ① • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?

Examples

- ① • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?

Examples

- ① • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?

Examples

- 1 • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- 2 • Consider the sequence:
13, 29, 53,

Examples

- 1 • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- 2 • Consider the sequence:
13, 29, 53,

Examples

- 1 • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- 2 • Consider the sequence:
13, 29, 53, 149,

Examples

- 1 • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- 2 • Consider the sequence:
13, 29, 53, 149, 173,

Examples

- ① • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- ② • Consider the sequence:
13, 29, 53, 149, 173, 269,

Examples

- 1 • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- 2 • Consider the sequence:
13, 29, 53, 149, 173, 269, 293,

Examples

- 1 • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- 2 • Consider the sequence:
13, 29, 53, 149, 173, 269, 293, 317, ...
What is the next number?

Examples

- 1 • Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- 2 • Consider the sequence:
13, 29, 53, 149, 173, 269, 293, 317, ...
What is the next number?
Can you express it in a “mathematical” sentence?

Examples

- ① Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- ② Consider the sequence:
13, 29, 53, 149, 173, 269, 293, 317, ...
What is the next number?
Can you express it in a “mathematical” sentence?
Can you formulate an hypothesis?

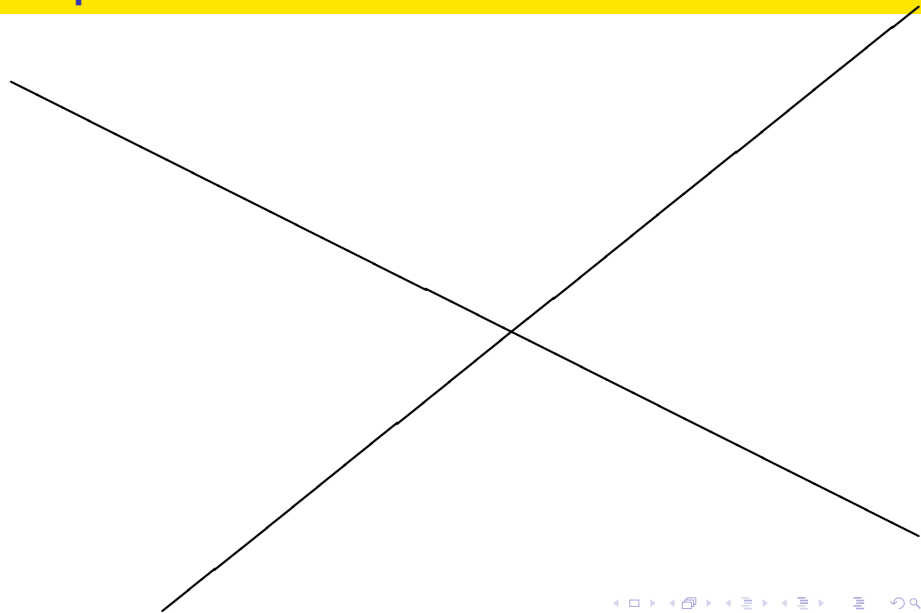
Examples

- ① Consider the sequence 1, 2, 4, 7, 11, 16, 22 ...
Can you express your observation in a “mathematical” sentence?
Does this sequence represent anything?
- ② Consider the sequence:
13, 29, 53, 149, 173, 269, 293, 317, ...
What is the next number?
Can you express it in a “mathematical” sentence?
Can you formulate an hypothesis?
Can you prove it?

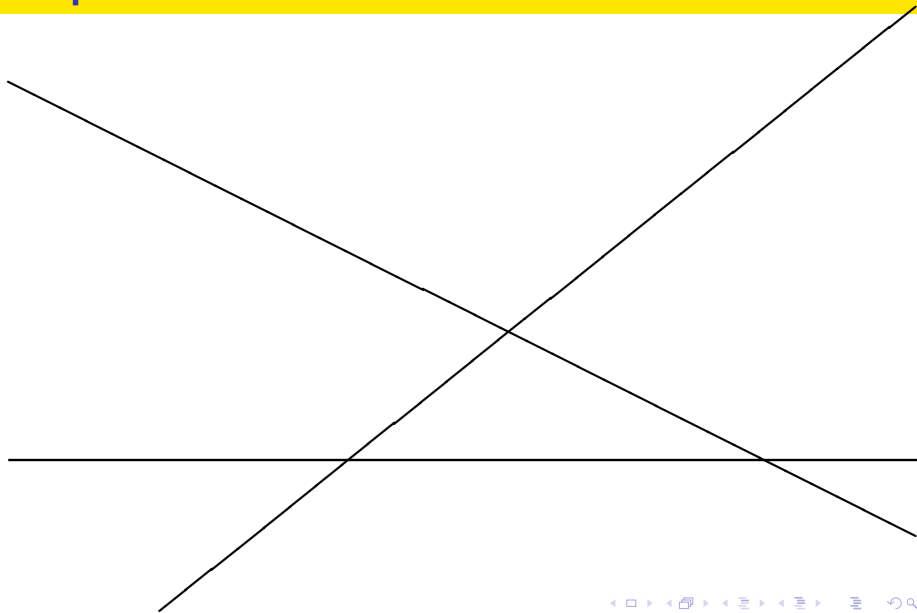
Shapes



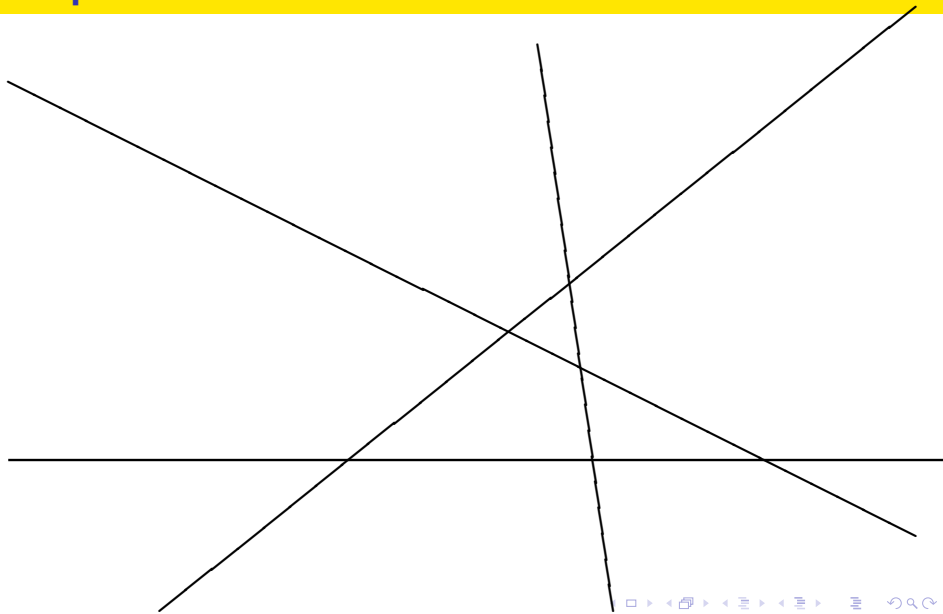
Shapes



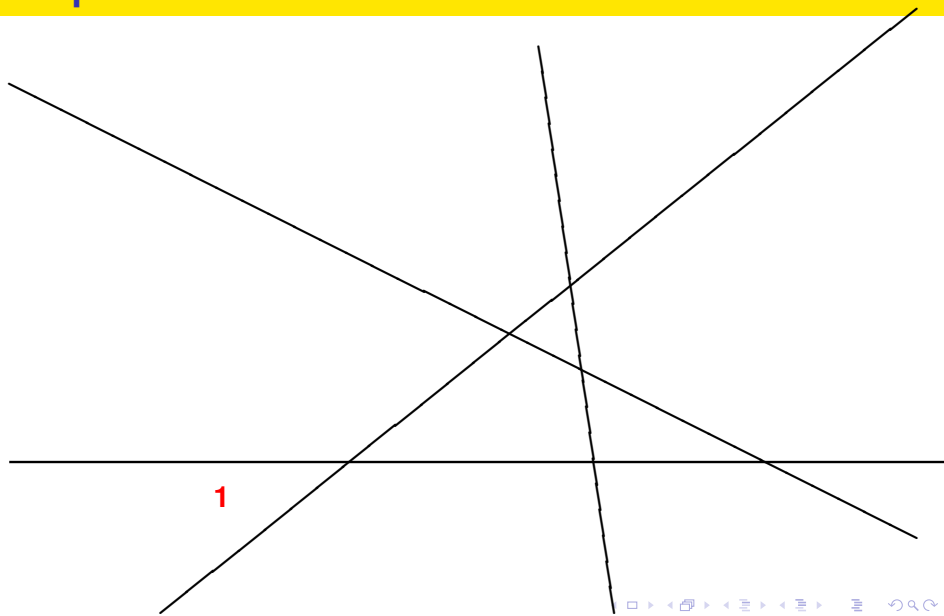
Shapes



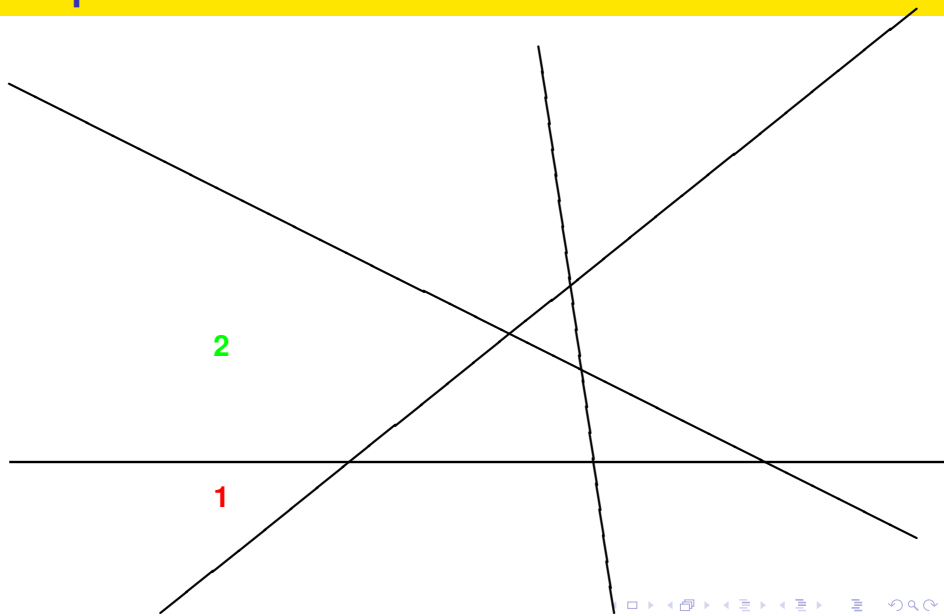
Shapes



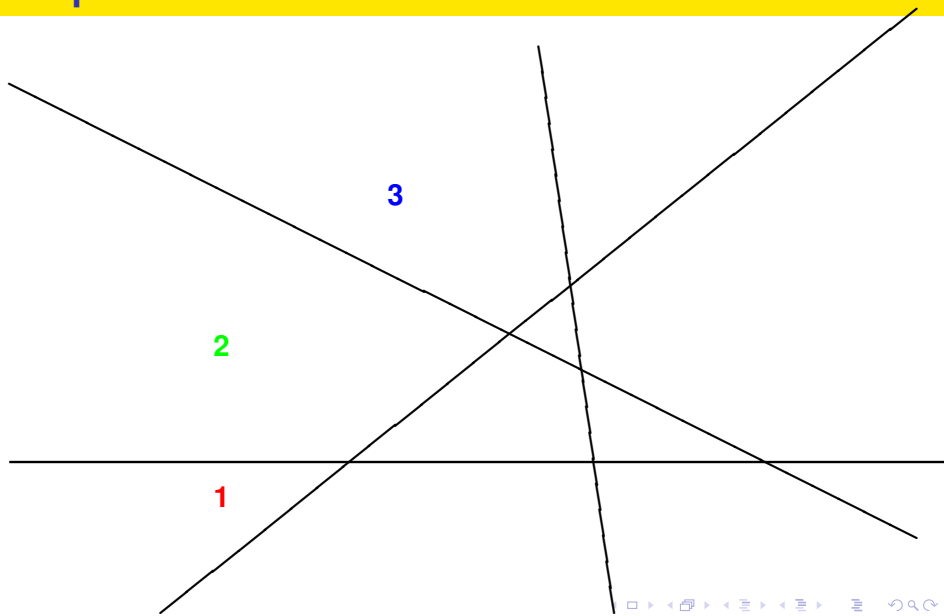
Shapes



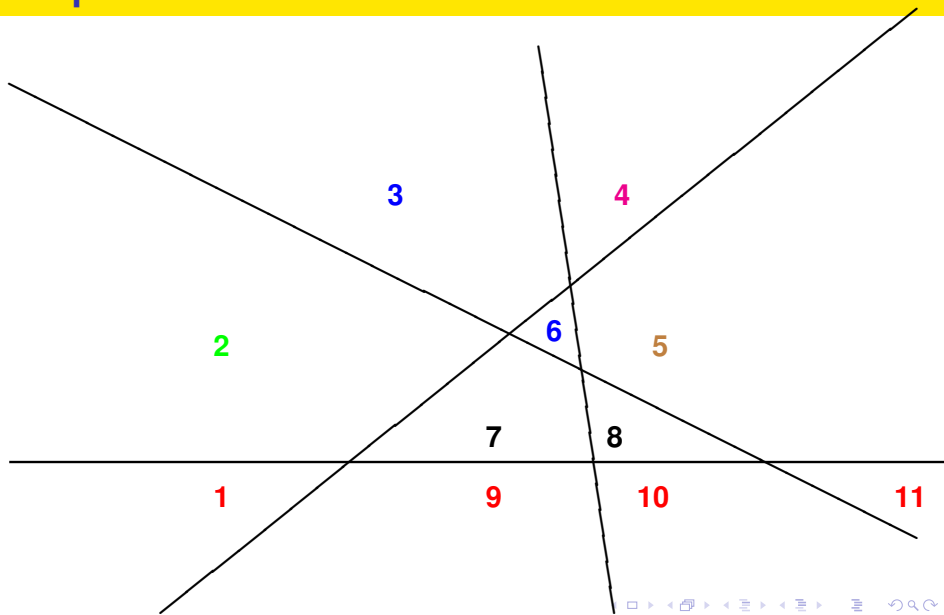
Shapes



Shapes



Shapes



Shapes

- ① How many regions can be generated by 5 lines?

Shapes

- 1 How many regions can be generated by 5 lines?
- 2 By 6 lines?

Shapes

- 1 How many regions can be generated by 5 lines?
- 2 By 6 lines?
- 3 Have you seen this sequence of numbers before?

Shapes

- 1 How many regions can be generated by 5 lines?
- 2 By 6 lines?
- 3 Have you seen this sequence of numbers before?
- 4 So can you tell how many regions can be generated by 17 lines?

Shapes

- 1 How many regions can be generated by 5 lines?
- 2 By 6 lines?
- 3 Have you seen this sequence of numbers before?
- 4 So can you tell how many regions can be generated by 17 lines?
- 5 Can you explain why?

Shapes

- 1 How many regions can be generated by 5 lines?
- 2 By 6 lines?
- 3 Have you seen this sequence of numbers before?
- 4 So can you tell how many regions can be generated by 17 lines?
- 5 Can you explain why?
- 6 Can you find a “formula” for $f(n)$ the number of regions generated by n lines?

Shapes

- 1 How many regions can be generated by 5 lines?
- 2 By 6 lines?
- 3 Have you seen this sequence of numbers before?
- 4 So can you tell how many regions can be generated by 17 lines?
- 5 Can you explain why?
- 6 Can you find a “formula” for $f(n)$ the number of regions generated by n lines?
- 7 Can you prove that it is $\binom{n+1}{2} + 1$?

Curiosity led research

You have four points on a line, the distance between any two consecutive points is **1**.

Curiosity led research

You have four points on a line, the distance between any two consecutive points is **1**.



❶ How many times does the distance **1** occur?

Curiosity led research

You have four points on a line, the distance between any two consecutive points is **1**.



- 1 How many times does the distance **1** occur?
- 2 How many times does the distance **2** occur?

Curiosity led research

You have four points on a line, the distance between any two consecutive points is **1**.



- 1 How many times does the distance **1** occur?
- 2 How many times does the distance **2** occur?
- 3 How many times does the distance **3** occur?

Curiosity led research

You have four points on a line, the distance between any two consecutive points is **1**.



- 1 How many times does the distance **1** occur?
- 2 How many times does the distance **2** occur?
- 3 How many times does the distance **3** occur?
- 4 Can you place n points in the plane so one distance will occur $n - 1$ times, a second distance will occur $n - 2$ times, a third $n - 3$ times etc.

Curiosity led research

You have four points on a line, the distance between any two consecutive points is **1**.



- 1 How many times does the distance **1** occur?
- 2 How many times does the distance **2** occur?
- 3 How many times does the distance **3** occur?
- 4 Can you place n points in the plane so one distance will occur $n - 1$ times, a second distance will occur $n - 2$ times, a third $n - 3$ times etc.
- 5 This was easy, can you do it so no three points are on a line?

Curiosity led research

You have four points on a line, the distance between any two consecutive points is **1**.



- 1 How many times does the distance **1** occur?
- 2 How many times does the distance **2** occur?
- 3 How many times does the distance **3** occur?
- 4 Can you place n points in the plane so one distance will occur $n - 1$ times, a second distance will occur $n - 2$ times, a third $n - 3$ times etc.
- 5 This was easy, can you do it so no three points are on a line?
- 6 This was not too difficult,

Curiosity led research

You have four points on a line, the distance between any two consecutive points is **1**.



- 1 How many times does the distance **1** occur?
- 2 How many times does the distance **2** occur?
- 3 How many times does the distance **3** occur?
- 4 Can you place n points in the plane so one distance will occur $n - 1$ times, a second distance will occur $n - 2$ times, a third $n - 3$ times etc.
- 5 This was easy, can you do it so no three points are on a line?
- 6 This was not too difficult,
- 7 but can you do it so the points will be in **general position** (no three on a line, no four on a circle)?