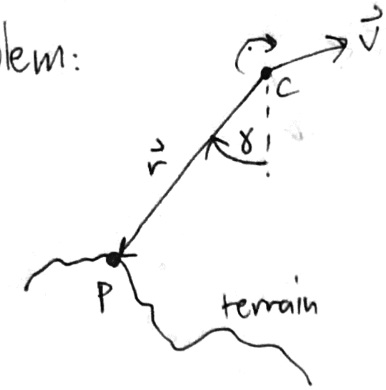
OPTIC FLOW

Problem:

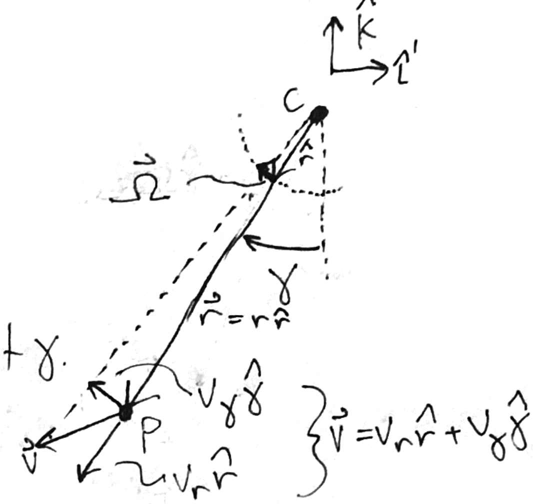


A camera at point C moves at velocity $\vec{v}$ and angular velocity ω. What is the "optic flow" caused by the terrain at point P, angle γ?

Solution: First consider a simpler problem:

C is fixed, P is moving at $\vec{v}$

define $\vec{\Omega} = \Omega\hat{\gamma}$ as the optic flow "vector". want $\Omega(\gamma)$ for point P at γ.



Note that $\frac{v_\gamma}{\Omega} = \frac{|\vec{r}|}{|\hat{r}|} = \frac{r}{1} \Rightarrow \Omega = \frac{v_\gamma}{r}$

Now, suppose that the camera is moving at $-\vec{v}$ and P is fixed. This is indistinguishable from "simpler" case.

$$\Rightarrow \Omega = -\frac{v_\gamma}{r}$$

Next, suppose the camera has $\vec{v} = 0$ but $\omega \neq 0 \Rightarrow \Omega = -\omega$

(note: effect of ω does not depend on r)

Sum these two effects to get $\Omega = -\omega - \frac{v_\gamma}{r}$

Component form: given $\vec{v}' = v_x'\hat{\imath}' + v_z'\hat{k}' = \begin{bmatrix} v_x' \\ v_z' \end{bmatrix}$, and $\hat{\gamma}' = \begin{bmatrix} -\cos\gamma \\ \sin\gamma \end{bmatrix}$

then $v_\gamma = \vec{v}\cdot\hat{\gamma} = -v_x'\cos\gamma + v_z'\sin\gamma \Rightarrow \boxed{\Omega(\gamma) = -\omega + \frac{v_x'}{r}\cos\gamma - \frac{v_z'}{r}\sin\gamma}$

Remark: in full 3D case, is 2D vector field on sphere $\vec{\Omega}' = -\vec{\omega}' \times \hat{r}' + \frac{1}{r}(I - \hat{r}'\hat{r}'^{T})\vec{v}'$