1. Consider again ionization, the process by which an atom or molecule acquires a positive or negative charge by gaining or losing electrons to form ions. A uniform electric field $\mathbf{E}\cos\omega t$ is applied to a hydrogen atom in its ground state. In each of the following parts, be sure to define all of your symbols except $\hbar$ and $c$.

(a). Hydrogen consists of an electron and proton bound together by the Coulomb force which depends on the distance between the electron and proton. Write the Hamiltonian and ground state wave function of hydrogen (in a general inertial reference frame) in terms of relative and center-of-mass coordinates.

(b) The final state consists of an electron and proton, in a continuum state which interacts via the same Coulomb force. Write the continuum wave function of the continuum state in a general inertial reference frame. Express your answer in terms of well-defined special functions.

(c) Compute the ionization rate of the atom as a function of $\omega$.

2. Two identical spin 1/2 fermions are subject to the Hamiltonian

$$H = H_0 + V, \quad H_0 = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2(r_1^2 + r_2^2), \quad V = V_0 \delta(r_1 - r_2), \quad V_0 > 0, \quad (1)$$

with $V$ to be regarded as a very small perturbation.

(a) Ignoring the effects of $V$ what would the ground state wave function and energy be?

(b) Compute the ground state energy, accurate to first-order in $V_0$.

(c) Consider the first two excited states, ignoring the effects of $V$. What would the wave functions and energies be?

(d) Include $V$ in first-order, to obtain compute the energies of the first two excited states.

3. Consider two electrons in an atom, each in a $p(l = 1)$ state. This configuration is denoted $p^2$. (a) Rewrite the configuration in terms of total orbital angular momentum $L$ and spin $S$: $2S + 1L$. Show that the allowed states are $^1S$, $^3P$, and $^1D$.

(b) The Coulomb interaction between electrons shifts the energy of the three configurations away from the unperturbed value $E_0$. Use first-order perturbation theory to show that

$$E(^1S) = E_0 + \langle \gamma_0 \rangle + \frac{10}{25} \langle \gamma_2 \rangle$$
$$E(^3P) = E_0 + \langle \gamma_0 \rangle - \frac{5}{25} \langle \gamma_2 \rangle$$
$$E(^1D) = E_0 + \langle \gamma_0 \rangle + \frac{1}{25} \langle \gamma_2 \rangle, \quad \langle \gamma_k \rangle = e^2 \int \int \frac{r_<}{r_{k+1}} | R(r) |^2 | R(r') |^2 r^2 dr r'^2 dr' \quad (2)$$

where $r_<$ is the lesser of $r, r'$, $r_>$ is the greater of $r, r'$, and in which $R(r)$ is the radial wave function normalized as $1 = \int r^2 dr \ | R(r) |^2$. Obtaining the coefficients of the terms $\langle \gamma_2 \rangle$ using involves a sum over Clebsch-Gordan coefficients.