Scattering of identical spin 0 bosons

\[ \vec{r}_1 \rightarrow \vec{r}_1' \text{ or } \vec{r}_2 \rightarrow \vec{r}_2' \text{ by } b \rightarrow \pi^+ \]

Review Scatt results For distinguishing particles \( r \rightarrow \infty \)

\[ \psi'(\vec{r}) = \eta \left[ \frac{\epsilon (\vec{k}_2 - \vec{v} \times \vec{k}_1) f(\vec{k}_1, \vec{k}_2)}{\sqrt{r}} \right] \]

\[ F = \vec{r}_1 - \vec{r}_2 \]

\[ k' = k - \frac{\vec{v}}{c} \cdot \vec{k} \]

\[ k' \cdot k = \cos \theta - \frac{k^2}{2} \]

\[ \frac{d\psi'}{dr} = \frac{\psi''}{d\psi / d\Omega} = \left| \frac{f}{f''} \right|^2 \]

For identical bosons spin 0

\[ \psi(\vec{r}) \rightarrow \psi(\vec{r}) + \psi(\vec{r}') \]

\[ + \vec{r} \rightarrow - \vec{r} \quad \vec{r}' \rightarrow - \vec{r}' \]

\[ f(\vec{k}_1, \vec{k}_2) \Rightarrow f(\vec{k}_1, \vec{k}_2) + f(-\vec{k}_1, \vec{k}_2) \]

\[ = f(\theta) + f(\pi - \theta) \]
\[ \frac{d\sigma^3}{d\ell} = \left( |f(0)|^2 + |f(r-\theta)|^2 \right) \text{ classical identity} \]

\[ + 2 \Re f(\theta) f^*(r-\theta) \]

Low energy - \( f(\theta) = \text{constant} \)

\[ \text{no Coul} \]

\[ \frac{d\sigma}{d\ell} = 4 \frac{d\sigma}{d\ell} \]
Physics example: Bromley et al. see GoH Fread p. 344

Two $^{12}$C nuclei:

Charge = $6e$

$E_{cm} = 5$ MeV

Carbon $^{12}$ has spin 0.

Treat as elem. particle.

1) Takes 16 MeV to break it up.

2) Coulomb field is smoothly varying over nuclear size $\propto$ just over nuclear size $\Rightarrow$ C is scattered.

3) $^{12}$C distance of closest approach:

$$\frac{2 \times 3 \times 6 e^2}{D} = 5 \text{ MeV}$$

$$D = \frac{5}{1.44 \text{ MeV fm}} = 3.6 \times 10^{-12} \text{ fm} = 10 \text{ fm}$$

$$r_{cm} = \varnothing 2.5 \text{ fm}$$

$^{16}$O

Electrons cancelled and far away.
\[
\left( -\nabla^2 + V_c \right) \psi = \frac{\hbar^2}{2\mu} \psi
\]

\[
V_c = \frac{2n^2 e^2}{r}
\]

\[
\nabla^2 + \frac{k^2}{r^2} + \frac{\kappa^2 e^2}{2\mu} \frac{2n}{r} \psi = 0
\]

\[
\psi = \frac{2n^2 e^2}{\kappa} \quad \hbar = \mu \kappa
\]

\[
\nabla^2 - \frac{k^2}{r^2} \psi = 0 = 0 \quad \Rightarrow \nabla^2 + 1 - \frac{k^2}{r^2} = \psi
\]

\[
E_{cm} = 5 \text{ MeV} = \frac{\hbar^2}{m} = \frac{2\mu^2 v^2}{2\mu} = \frac{2\mu v^2}{2} \frac{F_{2}(p)}{G_{2}(p)}
\]

\[
\frac{\mu}{2} = \frac{M_{e}}{2}
\]

\[
u^2 = 10 \text{ MeV}
\]

\[
u^2 = \frac{4.5 \text{ MeV}}{12,940 \text{ MeV}} = 0.042
\]

\[
\psi = \frac{36}{137} \left( \frac{1}{0.042} \right) > 6.3 \quad \sin \left( \frac{\pi \theta}{2} \right) = \cos \theta/2
\]

\[
f_{c} = \frac{\psi}{2 \times \sin \theta}
\]

\[
d\alpha = \frac{\psi}{\sinh} \left[ \frac{1}{\sinh} + \frac{1}{\cosh} + \frac{2 \Re}{\sin \theta \cos \theta} \right] \left( \frac{1}{\sinh} + \frac{1}{\cosh} + \frac{1}{\sin \theta \cos \theta} \right) \cos \left( 2 \times \arctan \theta \right)
\]
\[ \frac{d\sigma}{d\Omega} \]

\[ 20 \ 40 \ 60 \ 80 \ 100 \ 120 \]

Bromley et al. PRL 4, 365 (1960)

\( ^{12}C + ^{12}C \) elastic

5 MeV cm
\[
f(\theta) = \frac{1}{2} \sum e^{\theta} \text{trunc} \quad \text{Pe} \text{coss}a \quad (2021)
\]

Now \[ F(\theta) + f(\theta, t_0) \quad \text{Pe} \text{coss}a \text{dil} \text{or Pe} \text{coss}a \]
\[
\text{Pe} \text{coss}a \text{dil} \quad z = (\text{-}1)^2
\]
\[
\frac{\partial}{\partial t} \quad F(t) = \sum e^{\theta} \text{trunc} \quad \text{Pe} \text{coss}a \text{dil}
\]
\[
\frac{\partial}{\partial t} \quad \frac{4}{\partial \theta} \quad \sum e^{\theta} \text{trunc} \quad \text{Pe} \text{coss}a \text{dil}
\]

\[ \text{ON L4 even} \]
Exclusion Principle

Consider many electron atom

\[ H = \sum \frac{p_i^2}{2m} - \sum \frac{2e^2}{r_i} + \frac{1}{2} \sum \frac{e^2}{r_{ij}} \]

(First approx - neglecting hyperfine splitting)

We will want to quant symmetrise but first must solve Schrodinger equation - dynamical aspect

Each electron feels attraction pull of towards the nucleus (Z+1 protons) but also a repulsion force interaction due to (Z-1) electrons

If one electron is far from atom \( Z \)

\[ \text{feels a potential } - \frac{e^2}{r_i} \quad (r_i > r_j) \]

At short distances there is a complex problem
\[ H = H + V - U \]
\[ = \sum_{i=1}^{2} \frac{p_i^2}{2m} - \sum_{i<j} \frac{2e^2}{\epsilon_{ij}} + \sum U_0 + \left( \sum \frac{e^2}{r_i} - U \right) \]

\[ H_0 = \sum \frac{p_i^2}{2m} - \sum \frac{2e^2}{\epsilon_{ij}} + U_0 \]

\[ H = H_0 + V, \quad V = \left( \sum \frac{e^2}{r_i} - U \right) \]

\[ H_0 = \sum_{i=1}^{2} \frac{p_i^2}{2m} \]

Eigenstates $|d\rangle$, $|e\rangle$, $|f\rangle$:

\[ H_0 \begin{pmatrix} d \rangle \end{pmatrix} = \epsilon_d \begin{pmatrix} d \rangle \end{pmatrix} \]

Degeneracy: Q of 2 if complete filling

\[ \Psi(n_1, n_2) \] represents $n_1, n_2$
There are $Z^3$ degenerate states

Take purely antisym. combination

3 body $Li(2+3) \quad 1s^1 1s^1 2p^2$

$$\left[ \begin{array}{c}
<1 1d> <2 1\beta> <3 1\beta> \\
<2 1d> <1 1\beta> <3 1\beta>
\end{array} \right]$$

$$+ \left[ \begin{array}{c}
<2 1d> <3 1\beta> <1 1\beta> \\
<1 1d> <2 1\beta> <1 1\beta>
\end{array} \right]$$

$$- \left[ \begin{array}{c}
<3 1d> <2 1\beta> <1 1\beta> \\
<1 1d> <3 1\beta> <2 1\beta>
\end{array} \right]$$

$$- \left[ \begin{array}{c}
<1 1d> <3 1\beta> <2 1\beta>
\end{array} \right] \frac{1}{\sqrt{6}}$$

$$\begin{array}{c}
<1 1d> <2 1\beta> <3 1\beta> \\
<1 1\beta> <2 1\beta> <3 1\beta>
\end{array}$$

$\sqrt{6}$

Slater Det

What to do for bosons?

In the case for any number of elect

contains 2 particle same state

Two same spin det are same

$\det = 0$

Pauli exclusion principle
Examples in Class

\[ \psi_2 = \frac{1}{\sqrt{2}} (|11\rangle \langle 22| \beta - |22\rangle \langle 11| \beta) \]

\[ \langle \psi_2 | H_0 | \psi_2 \rangle = 3 \alpha + 3 \beta \]

\[ \langle \psi_2 | \frac{e^2}{2} | \psi_2 \rangle = \langle \alpha \beta | v \alpha \beta \rangle \]

\[ \gamma \]

\[ - \langle \alpha \beta | v \alpha \beta \rangle \]

\[ = \langle \alpha \beta | v \alpha \beta \rangle \]