PHYSICS 323 ELECTROMAGNETISM

May 26, 2020 Problem Set 7 These 4 problems are due in Canvas by 11 am on **Thursday**, June 4

Please put your name and section number on the first page of your solutions. The graded homework will be returned in your sections.

1 *Threshold kinetic energy*

The threshold energy for a reaction in which a projectile plus a target at rest makes a given final state is the minimum kinetic energy for the reaction to occur. This is an important quantity if one wants to discover new particles. Calculate the threshold kinetic energy for each of the following two reactions in which a pion is incident on a proton at rest. The rest energies (Mc^2) are $\pi(140)$, $p(940)$, $\Lambda(1115)$ $K(495)$ in units of MeV.

(a) $\pi + p \to \Lambda + K$

(b) $\pi + p \to p + K + K$

2. *Electron-positron annihilation*

A positron with kinetic energy equal to twice its rest mass energy is incident on an electron at rest. The positron and electron annihilate creating two photons. One photon goes off at an angle of 30° with respect to the incident positron. Compute the energies of the two photons and find the direction in which the second photon travels.

3. *Transforming fields*

(a) Show that the quantities $\mathbf{E} \cdot \mathbf{B}$ and $E^2 - c^2B^2$ are invariant under Lorentz transformations.

(b) The magnetic field of the earth is predominantly vertical in the polar regions. It was suggested to use this field to measure the speed of an aircraft by measuring the voltage induced in a wire suspended in the aircraft. What orientation should the wire have relative to the aircraft? Estimate the magnitude of the voltage induced in a 1 m wire.

4. *Rank-2 tensors* A charge q moves with a constant velocity $\mathbf{u} = u\hat{\mathbf{x}}$. Define the antisymmetric tensor $r^{\mu\nu}$ by $r^{\mu\nu} \equiv \frac{1}{c}(\eta^\mu x^\nu - \eta^\nu x^\mu)$, where η^μ is the four velocity of the charge. Further define $r^2 = -r^{\mu\nu}r_{\mu\nu}/2$.

(a) Show that $r^2 = \frac{(x-ut)^2}{1-u^2/c^2} + y^2 + z^2$.

(b) Show that the electric field produced by the charge is given by $E_n/c = \frac{q}{4\pi\epsilon}\frac{r^{0n}}{(r^2)^{3/2}}$, where the subscript n refers to the x, y or z component of $\mathbf{E}$. It is sufficient to show that the above expression is equivalent to one derived in the lectures or in the textbook.