PHYSICS 323 ELECTROMAGNETISM

May 17, 2020 Problem Set 6 These 4 problems are due **in to Canvas 11 am on Tuesday, 26 May.**

Please put your name and section number on the first page of your solutions.

1 *Frequency dependence of radiation* The angular distribution of radiation, the power emitted per sold angle is $\frac{dP}{d\Omega} = r^2 \mathbf{S}\cdot\hat{\mathbf{r}}$, where $\mathbf{S}$ is the Poynting vector. $\mathbf{S}(\mathbf{r},t) = \frac{1}{\mu_0 c}\hat{\mathbf{r}}\,\mathbf{E}(\mathbf{r},t)\cdot\mathbf{E}(\mathbf{r},t)$ for observer's positions and times $(\mathbf{r},t)$ in the radiation zone. The aim of this problem is to understand the frequency dependence of any emitted radiation. First, recall the Fourier transform, for example $\mathbf{E}(\mathbf{r},t) = \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{2\pi}}\mathbf{E}(\mathbf{r},\omega)e^{-i\omega t}$. The inverse is effected using the delta function:$\delta(\omega-\omega') = \int_{-\infty}^{\infty}\frac{dt}{2\pi}e^{i(\omega-\omega')t}$, so that $\mathbf{E}(\mathbf{r},\omega) = \int_{-\infty}^{\infty}\frac{dt}{\sqrt{2\pi}}\mathbf{E}(\mathbf{r},t)e^{i\omega t}$. The total energy radiated per solid angle $\frac{dU}{d\Omega} = \int_{-\infty}^{\infty} dt\,\frac{dP}{d\Omega}$.

(a) Show that $\frac{dU}{d\Omega} = r^2\epsilon_0 c\int d\omega\mathbf{E}(\mathbf{r},\omega)\cdot\mathbf{E}^*(\mathbf{r},\omega)$.

(b) Show that in the Lorentz gauge the frequency-dependent vector potential $\mathbf{A}(\mathbf{r},\omega)$ can be expressed in the radiation zone as $\mathbf{A}(\mathbf{r},\omega) = \frac{\mu_0}{4\pi r}e^{ikr}\mathbf{J}(\mathbf{k},\omega)$, where $k=\omega/c$ and
$\mathbf{J}(\mathbf{k},\omega) = \int d^3r'dt'/\sqrt{2\pi}\mathbf{J}(\mathbf{r}',t')e^{-i\mathbf{k}\cdot\mathbf{r}}e^{i\omega t'}$, where $\mathbf{k}\equiv k\,\hat{\mathbf{r}}$.

(c) One may write $\frac{dU}{d\Omega} = \int_{-\infty}^{\infty} d\omega\frac{d^2U}{d\omega\delta\Omega}$, and define $\frac{dI}{d\Omega}\equiv\frac{d^2U}{d\omega\delta\Omega}$, The angular distribution of energy per solid angle per frequency. Show that $\frac{dI}{d\Omega} = \frac{c\mu_0}{16\pi^2}\left[k^2\mathbf{J}(\mathbf{k},\omega)\cdot\mathbf{J}^*(\mathbf{k},\omega) - (\mathbf{k}\cdot\mathbf{J}(\mathbf{k},\omega))(\mathbf{k}\cdot\mathbf{J}^*(\mathbf{k},\omega))\right]$.

2. *Time and space*

A rocket ship leaves earth at a speed of $2/5c$. When a clock on the rocket says 2 hours have elapsed, the rocket ship sends a light signal back to earth.

(a) According to clocks on the earth, when was the signal sent?

(b) According to clocks on the earth, how long after the rocket left did the signal arrive back at earth?

(c) According to an observer on the rocket, how long after the rocket left did the signal arrive back on earth?

3. *Simultaneous?* In a reference frame S two very evenly matched sprinters are lined up a distance L apart along the x axis for a race parallel to the y axis. Two starters, one beside each runner will fire their pistols at lightly different times, giving a handicap to the faster of the two sprinters. This means that in the frame S, the faster of the two sprinters starts later by a time difference $T>0$.

(a) For what range of time differences T will there be a reference frame $\bar{S}$ in which there is no handicap, and for what range of time difference is there a true (not apparent) handicap.

(b) Determine the Lorentz transformation to the frame $\bar{S}$ appropriate for each of the two possibilities in part (a), finding the velocity of $\bar{S}$ relative to S .

4. *Two Lorentz transformation* Show that two successive Lorentz transformations with relative velocities in the same direction v_1 and v_2 relative to a given inertial reference frame are equivalent to a single Lorentz transformation and determine the velocity parameter of that single Lorentz transformation.