

PHYSICS 323 ELECTROMAGNETISM

May 3, 2020 Problem Set 4 These 4 problems are due in Canvas Tuesday, May 12 11 am

Please put your name and section number on the first page of your solutions.

1. Antenna problem

A wire of length L has its center at the origin, and lies along the z axis. The current carried by the wire is of the form $I(z, t) = I_0 \cos(\omega t) \cos(\frac{\pi z}{L})$. Consider positions in the radiation zone.

- Compute $\mathbf{A}(\mathbf{r}, t)$.
- Compute $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$.
- Compute the angular distribution of radiated power.

2. Magnetic dipole radiation- two loops

Consider a circular current loop in the xy plane of radius b as in Section 11.1.3. Suppose there is another parallel identical loop placed a distance d above the first loop. The directions of the currents in the two loops are both counterclockwise. The long wavelength approximation is applicable.

- Compute the total vector potential \mathbf{A} at positions in the radiation zone.
- Compute the total magnetic field in the radiation zone.
- Compute the angular distribution of radiated power, and discuss the dependence on the value of d .

3. Pulsar

A pulsar is a spinning neutron star where the star's magnetic-dipole axis is misaligned by angle α with its spin axis. The equation for the Poynting vector (11.39) is modified by including a term $\sin^2 \alpha = 1/2$. A typical neutron star has radius 10 km, and the magnetic field at the pole is 10^{12} Gauss.

- The famous Crab pulsar has a period of 33 ms. Estimate how much power it is radiating.
- The rarer magnetar has pole field or around 10^{15} Gauss and suppose the period is 1 ms. Estimate how much power it is radiating.

4. *Larmor radiation* A beam of 2 keV electrons is stopped with constant deceleration in a distance of 0.01 cm.

- Compute the total energy of radiation emitted by each electron.
- Determine the ratio of the emitted energy to the initial electron energy. Use this ratio to find the radiated power for an X-ray machine with 300 watts of power input to accelerate the electrons.
- Compute the maximum intensity (in Watts/meter²) of the radiation at a distance 20 cm from the stopping target.