PHYSICS 323 ELECTROMAGNETISM

April 21, 2020 Problem Set 3 These 4 problems are due **in Canvas Tuesday, 28 April.**

Please put your name and section number on the first page of your solutions.

1. *Review of Delta functions*

(a) Show that the integral: $\int_0^\pi \delta\left(x - \cos x\right) dx = \frac{1}{1+\sin x_0}$, where x_0 is the solution of the transcendental equation $x_0 - \cos x_0 = 0$. Then obtain a numerical answer.

(b) Show that $(\nabla^2 + k^2)\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = -4\pi\delta(\mathbf{r}-\mathbf{r}')$

(c) The time-averaged electric potential of a neutral hydrogen atom is given by

$$V(r) = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}(1 + \frac{\alpha r}{2}),$$

where q is the magnitude of the electronic charge and $\alpha^{-1} = a_0/2$, where a_0 is the Bohr radius. Find the charge distribution (continuous and discrete) that yields this potential and interpret your result.

2. *Time dependent point charge*

A time-dependent point charge $q(t)$ at the origin $\rho(\mathbf{r}, t) = q(t)\delta(\mathbf{r})$ is fed by a current $J(\mathbf{r}, t) = -\frac{1}{4\pi}(\dot{q}/r^2)\hat{\mathbf{r}}$

(a) Show that charge is conserved by confirming that the continuity equation (5.29) is obeyed.

(b) Determine the non-unique scalar and vector potentials in the **Lorentz** gauge.

(c) Find **E** and **B**

3. *Potentials and Gauges* The Coulomb gauge, defined by $\boldsymbol{\nabla} \cdot \mathbf{A} = 0$, is useful in atomic physics. The following questions refer to using the Coulomb gauge.

(a) Find the partial differential equation satisfied by the vector potential **A**. This equation contains the scalar potential V.

(b) In your result for (a): eliminate the term containing V by separating the current density **J** into two terms $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$ where $\boldsymbol{\nabla} \times \mathbf{J}_1 = 0$ and $\boldsymbol{\nabla} \cdot \mathbf{J}_2 = 0$, so that the dependence on V is eliminated and the equation for **A** contains only $\mathbf{J}_2$. The names for the separate currents are $\mathbf{J}_1$ is longitudinal component and $\mathbf{J}_2$ is transverse component. Thus **A** will depend only on $\mathbf{J}_2$

4. *Current in a long-straight wire*

(a) Suppose that a current in a long straight vertical wire carries a current that increases linearly with time: $I(t) = I_0\frac{t}{\tau}$ where I_0 is a constant with dimension of current, and τ is a constant with dimensions of time. Find the electric and magnetic fields that are generated.

(b) Do the same for a sudden burst of current $I(t) = q_0\delta(t)$.