PHYSICS 323 ELECTROMAGNETISM

6 April 2019 Problem Set 2 These problems are due **in Canvas l by 11 am on Tuesday, 16 April.**

Please put your name and section number on the first page of your solutions.

1. *Reflection from a conductor at an oblique angle*

(a) Begin by showing that the left-hand part of eq (9.109) can be written as $\tilde{E}_{0_R} = \frac{\tan(\theta_T - \theta_I)}{\tan(\theta_T + \theta_I)} \tilde{E}_{0_I}$

(b) Consider light, of angular frequency ω, incident on a conductor at an angle θ_i with respect to the normal. Both media have ϵ_0, μ_0 and the conductivity of the conductor is σ. Show that the wave number of the transmitted light, $\tilde{k}$ can be written as $\tilde{k} = \frac{\omega}{c}\tilde{n}$ and identify $\tilde{n}$

(c) Show that $\sin\theta_I = \tilde{n}\sin\theta_T$, where θ_T is the angle of refraction. Is θ_T a real number?

(d) Show that, for propagation of light in the conductor, the planes where the amplitude of the transmitted wave is constant are those of constant z, where the coordinate system is defined in Fig.9.15.

(e) Identify the the surfaces of constant real phase. Are these the same as the surfaces of constant z?

2. *Electromagnetic waves on skin* So far we have discussed reflection and transmission for dielectrics, for which $\frac{\epsilon}{\sigma}\omega \approx 0$ and for conductors for which $\frac{\epsilon}{\sigma}\omega = \omega\tau \gg 1$. But many interesting systems are intermediate between these extremes.

(a)For biological tissues with high water content, including skin, ϵ and σ depend on frequency. At $\nu = 2450$ MHz their values are $\epsilon = 47\epsilon_0$ and $\sigma = 2.21\Omega^{-1}\mathrm{m}^{-1}$. Determine $\omega\tau$ and the attenuation length, the distance for the intensity of the wave to drop by a factor of e.

(b) If a radar signal at 2450 MHz impinges on skin and other similar tissues, from air, how much of the incident power is absorbed? Assume the tissue thickness is 15 cm.

3. *Complex Wave Impedance* (a) Consider an electromagnetic wave traveling in free space with electric field E and magnetic field B. Determine the value of E/H where E and H are magnitudes of the field. This ratio is defined to be the wave impedance of free space.

Now consider plane waves of angular frequency ω normally incident on a non-magnetic good conductor of high conductivity σ. (b) Determine the complex wave impedance of the metal.

(c) Determine the reflection coefficient in terms of the wave impedance of free space and the wave impedance of the metal.

4. *Refraction and Dispersion of Hydrogen* Model the hydrogen atom as a point nucleus with charge e enveloped by a uniformly charged spherical electron cloud with a radius a_0 and total charge $-e$.

(a) If the point nucleus and the center of the electron cloud are separated by a distance d ($d \ll a_0$), determine the force (magnitude and direction) on the nucleus.

(b) What is the net force (magnitude and direction) on the electron cloud?

(c) The mass of the nucleus (m_p) exceeds the mass of the electron (m_e) by about 1840 making its motion negligible compared to that of the electron. Given the form of the force that you found in (b), describe the motion of the electron.

(d) Determine the numerical value of the frequency associated with the electron motion. You will need to look up the numerical values of m_e, e, and a_0 (Bohr radius).

(e) Deterimine the vacuum wavelength associated with this frequency for electromagnetic (em) radiation? Comment on where this lies in the em spectrum (visible light or radio-frequency or gamma ray etc).

(f) Find the coefficients of refraction and dispersion (eq 9.174) for this primitive model of the hydrogen atom. Assume that macroscopic system has N hydrogen atoms per unit volume.