PHYSICS 323 ELECTROMAGNETISM

30 March 2020 Problem Set 1 These problems are due **in CANVAS by 11 am on Tuesday, 7 April. Please put your name and section number on the first page of your solutions.**

1. *Transmission through glass* Consider light traveling from $x = -\infty$, incident normally on a plate of glass of thickness a. The plate is parallel to the yz plane, with one face at $x = 0$ and the other at $x = a$. The index of refraction is unity for $x < 0$ and $x > a$, and $n = 1.5$ for $0 \le x \le a$. The electromagnetic field in the region $x < 0$ is a superposition of right and left traveling waves which are the incident and reflected waves. In the region $0 \le x \le a$ there are both right and left traveling waves, and in the region $x > a$ there is only the transmitted right traveling wave.

(a) Write $\mathbf{E}(x, t)$ and $\mathbf{B}(x, t)$ in the three regions, letting the polarization of the incident wave be in the y direction. Write the four boundary conditions on the wave amplitudes.

(b) Determine the transmission coefficient T, the ratio of transmitted intensity to incident intensity.

(c) Plot T as a function of ka, where k is the incident wave number.

2. *Pulse of radiation moving in the x direction.* Consider the fields

$$E(\mathbf{r}, t) = \hat{\mathbf{i}}F_1(x - ct) + \hat{\mathbf{j}}F_2(x - ct) + \hat{\mathbf{k}}F_3(x - ct),\ cB(\mathbf{r}, t) = \hat{\mathbf{i}}G_1(x - ct) + \hat{\mathbf{j}}G_2(x - ct) + \hat{\mathbf{k}}G_3(x - ct),$$

where the functions F_i, G_i approach 0 in the limits $x \to \pm\infty$. These fields satisfy the wave equation and correspond to a pulse of radiation moving in the $+x$ direction. The Maxwell equations place severe restrictions on the components F_i, G_i.

(a) Show that the Maxwell equations require that $F_1 = G_1 = 0$, $G_3 = F_2$ and $G_2 = -F_3$.

(b) Suppose $F_2(\xi) = G_3(\xi) = E_0 \exp(-\xi^2/a^2)$ and other components are 0. Make a sketch that shows a snapshot of the electric and magnetic fields at a time t.

3. Polarization perpendicular to the plane of incidence A plane electromagnetic wave is incident from one linear medium to another.

(a) Impose the boundary conditions and obtain Fresnel equations for $\tilde{E}_{0_R}$ and $\tilde{E}_{0_T}$.

(b) Compute the fraction of incident energy that is reflected.

(c) For incident light polarized in a direction parallel to the plane of incidence the fraction of incident energy that is reflected is denoted $R_{\|}$. For perpendicular polarization the fraction reflected is denoted: $R_\perp$. Suppose the incident medium is air $n = 1$ and the second medium has an index of refraction close to 1. Furthermore the angle of incidence is nearly grazing $\theta_I \approx \pi/2$. Show that $R_{\|} \approx R_\perp$ and derive each to first-order in small quantities.

4. *Applying Maxwell's Equations*

(a) Can $\mathbf{B} = B_0 \cos ax\, \mathbf{i}$ be a solution of Maxwell's equations? Explain your answer.

(b) Consider a region of free space that contains no free charges and currents. Can $\mathbf{H} = H_0 \cos(\beta z)\mathbf{j}$ be a solution of Maxwell equations? The answer is yes or no, but to get credit you must use Maxwell's equations to explain your answer.

(c) A capacitor with circular parallel plates, with radius a and separation $d \ll a$, has potential difference $V(t)$. Determine the magnitude of the magnetic field B on the midplane of the capacitor at a distance $s > a$ from the symmetry axis.

(d) Suppose the material between the capacitor plates was a medium with non-zero conductivity σ. Explain how your answer to part (c) would be be changed.