Name ______________________________ **Student ID** ____________ **Score**________
last *first*

IV. [20 points total] Tutorial question.

A positive point charge oscillates sinusoidally along the *x*-axis between the origin and $x = 1$ lightsecond (ls) every 4 seconds, such that it is at the origin when $t = \{0\text{s}, 4\text{s}, 8\text{s}, \dots\}$ and at $x = 1$ ls when $t = \{2\text{s}, 6\text{s}, 10\text{s}, \dots\}$.

A. Consider a location along the *x*-axis at $x = 6$ ls at the instant $t = 10$s.

i. [6 pts] What is the retarded time from the point charge to the location at this time? If the exact retarded time cannot be determined, give an estimate by rounding to the nearest second. Explain your reasoning.

4 seconds. (5 seconds is acceptable)

We have the equation $t_{ret} = t - \frac{|r-r'|}{c}$*, but the interpretation is key. The point of retarded time is backtracking the clock time t = 10 s to when the information was created based on the distance* $|r - r'|$*. Thus we must find where the point charge* ***was*** *in order to send the information to arrive at x = 6 ls at t = 10s.*

Since the point charge oscillates between x = 0 and 1, we know that $|r - r'|$ *is between 5 and 6 ls, so the possible retarded times is between 4s and 5s, inclusive. We can see that evaluating the point charge's location at* t_{ret} *= 4s does give the correct relationship: 6 seconds later due to the time delay, location x = 6 seconds receives the information at clock time 10 s.*

N.b., The correct formulation of retarded time is as described above: the distance measurement must describe the point of interest measured at that clock time, but the source's location at the retarded time. Due to the vagueness of the prompt "from the point charge," 5 seconds will also be accepted if you calculated a 5 s delay based on the charge being at x = 1 ls and show that 10 – 5 = $\boxed{5}$.

ii. [6 pts] Compare the Coulomb scalar potential V_{Coulomb} to the Lorentz scalar potential V_{Lorentz} at this location and time ($x = 6$ ls and $t = 10$s). Is V_{Coulomb} *greater than, less than, or equal to* V_{Lorentz} at this location and time? Explain your reasoning.

Greater than.

The Coulomb potential is instantaneous and uses the charge's location at the clock time t = 10s, but the Lorentz potential accounts for propagation of information by using the charge's location at the retarded time. At t = 10s, the point charge is closest to the location x = 6 ls, so the Coulomb potential is maximal. At t_{ret} *4s or 5s, the point charge is at the origin or at x = ½ ls, respectively, so the Lorentz potential is smaller due to the larger distance to the location x = 6 ls.*

B. [8 pts] Consider points very far from the origin along the *x*-axis. To what power of *x* is the leading order of the time-averaged electric field? *(I.e.,* The electric field depends mostly on x^n; what is *n?)* Explain your reasoning.

-2 or $\frac{1}{x^2}$

In terms of behavior far from the origin: radiation is 1/r, point charge field is $1/r^2$*, dipole is* $1/r^3$*, etc.. Thus in general, the radiative field is more important than the point charge field very far from the origin.*

However, radiation has a $\sin^2(\theta)$ *term where θ is the angle measured relative to the axis of the acceleration. Thus along the x-axis, the radiative field is zero!*

This means that the leading order of the electric field along the x-axis is from the point charge term, so $\frac{1}{x^2}$.