Begin with quick review of what you've seen in other courses

Theory of relativity

Relativity in physics refers to relative motion, not a theory of families

(1) Same laws of physics apply in any intertial ref frame (IRF)

IRF is one in which ~~objects~~ if there is no force on an object in moves with constant $\vec{v}$ or remains at rest

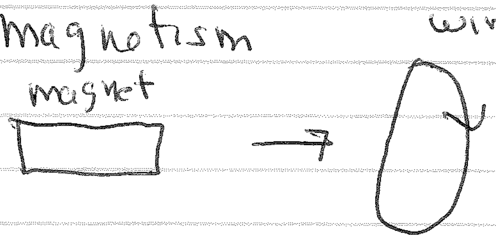
Thus an IRF moves with constant $\vec{v}$ or is at rest.

This idea is a basic part of mechanics

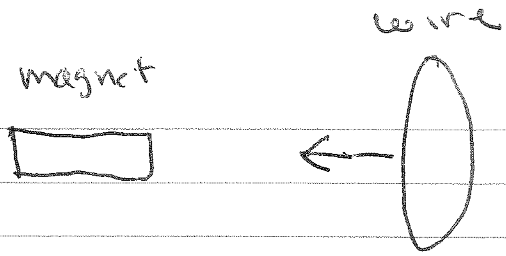
$\vec{F} = m\frac{d\vec{v}}{dt}$ if $\vec{F} = 0$, $\frac{d\vec{v}}{dt} = 0$, $\vec{v} = \overrightarrow{\text{constant}}$

In a frame moving with $\vec{v}_0$ (constant) $\vec{v} \to \vec{v} + \vec{v}_0$, $\frac{d\vec{v}}{dt}$ not changed

We have seen an example of relativity in magnetism



moving magnet make $\vec{B}(t)$ and $\mathcal{E} = -\frac{d\Phi}{dt} \Rightarrow$ current



If wire moves electrons experience a

force $\vec{F} = q\vec{v} \times \vec{B}$ and a current exist

$\mathcal{E} = -\frac{d\Phi}{dt} \rightarrow I$ Same result occurs

Electricity & Magnetism

Maxwell's equations work in any IRF

postulate 1.

Second aspect concerns speed of light in

free space

$$\left\{ \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right\} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix} = \overrightarrow{Sources}$$

This is a wave equation for EM wave

moving with speed c $= \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

(2) Second postulate of relativity
This is independent of the speed of the source.

Einstein postulate - universal speed of light in vacuum. The speed is c independent of source

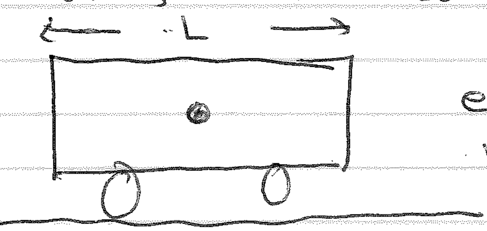
Truly amazing - great step forward

speed Usually depends on motion of the frame and motion of source of wave -

Many consequences of postulate 2

Meaning of simultaneity is frame-dependent

In train moving with constant v to right



emit light

moves with speed c

In train time to hit the front = $L/2c$

two events light emitted at initial time from middle of train; second event, hits front timespace

Observer on ground sees front moving to right while light catches up

how long till light hits front T_F

D light travels distance $D = \frac{L}{2} + vT_F$

$$T_F = \frac{D}{c} = \frac{L}{2c} + \frac{vT_F}{c} \; \rightarrow \; T_F = \frac{L/2c}{1 - v/c}$$

Meanwhile the back of train is moving toward

Source of light   Light moves $D_B = \frac{L}{2} - vT_B$

$$T_B = D_B/c = L/2c - V/c\, T_B$$

$$T_B = \frac{L/2c}{1-V/c}$$

$\underline{T_F < T_B}$!

events not simultaneous

Whether or not two events are
Simultaneous depends of IRF

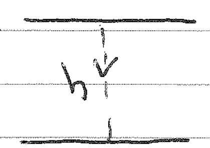
Another ~~...~~ consequence - time dilation

(dilation means becoming wider)

Say you are in a moving train v

event 1 emission at top at initial time

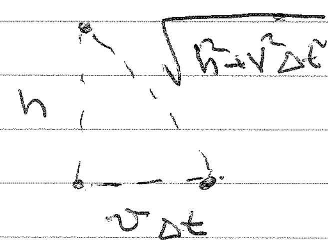
Send light from ceiling to floor



event 2 hits bottom later

in train $\Delta\tilde{t} = h/c$

Same two events seen from ground



$$\Delta t = \frac{\sqrt{h^2+v^2\Delta t^2}}{c}$$

Eq for Δt, square to find

$$\Delta t = \frac{h}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$

$\beta = V/c < 1$ $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} > 1$

$\Delta t = \gamma\Delta\tilde{t}$, $\Delta t > \Delta\tilde{t}$

Spectacular confirmation

of this almost all particles decay.

all $n \to p\, e^- \bar{\nu}$

muon $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

muon at rest Decay time = average decay time

$= 2 \times 10^{-6}$ s

studied many times μ^- atoms

beam Fermi Lab

$1 - (v/c)^2 \approx 10^{-8}$ $\gamma = 10^4$

lasts in Lab $2 \times 10^{-6}\,s\; \gamma$

$= 2 \times 10^{-2}$ s | $D = vt$

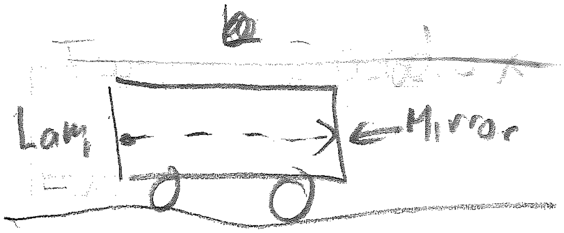
$= 3 \times 10^8$ m $\times 2 \times 10^{-2}$

$= 6 \times 10^6$ m

= Name

$\boxed{\text{time dilation}}$

Length



Compute time to hit mirror and return in train

$$\Delta \tilde{t} = \frac{2\Delta \tilde{x}}{c}$$ In train

Ground — front end moves away
rear end moves towards

time to hit front

$$\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c}$$

$$\Delta t_1 = \frac{\Delta x/c}{(1-v/c)}$$

$$\Delta t_2 = \frac{\Delta x - v\Delta t_2}{c}$$

$$\Delta t_2 = \frac{\Delta x/c}{1+v/c}$$

$$\Delta t = \Delta t_1 + \Delta t_2 = \Delta x/c \left[\frac{2}{1-v^2/c^2}\right] = \gamma^2\, 2\frac{\Delta x}{c}$$

$$\Delta t = \gamma \Delta \tilde{t} = 2\gamma \frac{\Delta \tilde{x}}{c} = 2\gamma^2 \frac{\Delta x}{c}$$

$$\Delta \tilde{x} = \gamma\, \Delta x$$ Length contraction

$$\Delta x = \frac{1}{\gamma} \Delta \tilde{x}$$

Both time dilation and length contraction

are summarized by the <u>Lorentz transformation</u> LT

~~Suppose we have two frames~~

We've seen several examples involving events

an event is something that occurs at a

specific time at a specific place

emission of light from here at initial time — later arrival here

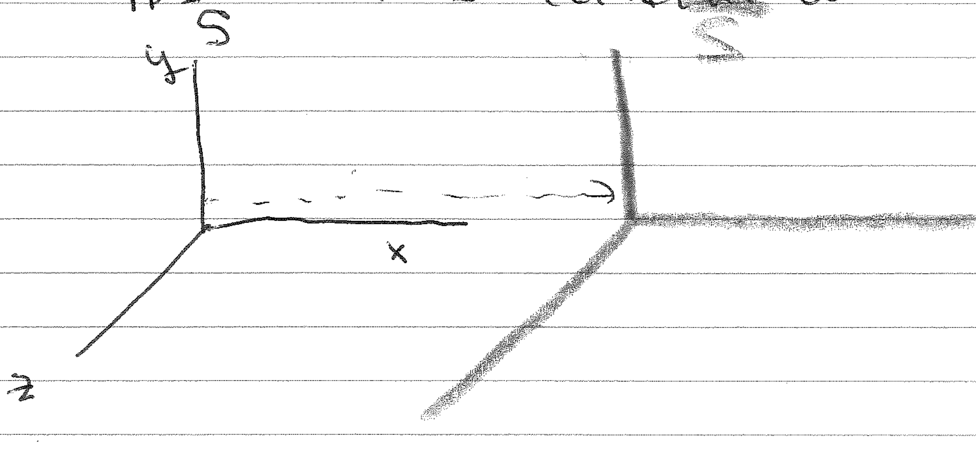
2 events

Suppose we have two inertial ref. frames $S, \bar{S}$
moving with relative velocity in x-direction

~~S has coordinates (x,y,z,t)~~

Events in S measured at (x, y, z, t)
" " $\bar{S}$ " " $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$

Suppose frames coincide at $t = 0$



The coordinates are related by

$$\bar{x} = \gamma\,(x - vt)$$
$$\bar{y} = y$$
$$\bar{z} = z$$
$$\bar{t} = \gamma\,(t - v/c^2\, x)$$

LT

LT accounts for time dilation and length contraction

Inverse LT $\rightarrow$ $v \rightarrow -v$
$\bar{x} \rightleftarrows x$ etc
$\bar{t} \rightleftarrows t$

$$x = \gamma\,(\bar{x} + v\bar{t}) \qquad t = \gamma\left(\bar{t} + \frac{v\bar{x}}{c^2}\right)$$

L. contraction

Stick moves to right with v

Length in $\bar{S}$ is $\bar{x}_R - \bar{x}_L = \Delta \bar{x}$

Observer in S measures left and right ends at one instant of her time

$$x = \gamma(\bar{x} + v\bar{t})$$
$$t = \gamma(\bar{t} + \frac{v}{c^2}\bar{x})$$

Length in S is Δx with $\Delta t = 0$

$$\Delta x = \gamma(\Delta\bar{x} + v\Delta\bar{t})$$
$$\Delta t = 0 = \Delta\bar{t} + \frac{v}{c^2}\Delta\bar{x} \rightarrow \Delta\bar{t} = -v/c^2\,\Delta\bar{x}$$
$$\Delta x = \gamma(\Delta\bar{x})(1 - \frac{v^2}{c^2}) = \frac{1}{\gamma}\Delta\bar{x}$$ Length contraction

What can we conserve

$$\Delta L^2 \equiv \Delta x^2 - c^2\Delta t^2$$
$$= \gamma^2(\Delta\bar{x} - v\Delta\bar{t})^2 - c^2\gamma^2(\Delta\bar{t} - \frac{v}{c^2}\Delta\bar{x})^2$$
$$= \Delta\bar{x}^2\gamma^2(1 - v^2/c^2) - \Delta\bar{t}^2c^2\gamma^2(1 - v^2/c^2)$$
$$= \Delta\bar{x}^2 - c^2\Delta\bar{t}^2$$ invariant interval

S ~~a measurement of separate at time same~~

Simultaneous measurements Length gives length in any frame $\Delta t = 0$ or $\Delta\bar{t} = 0$ give length. Similarly measurements the deal made at ~~and~~ same places is invariant

ΔL^2 describes a spacetime interval

if $\Delta L^2 > 0$ space like

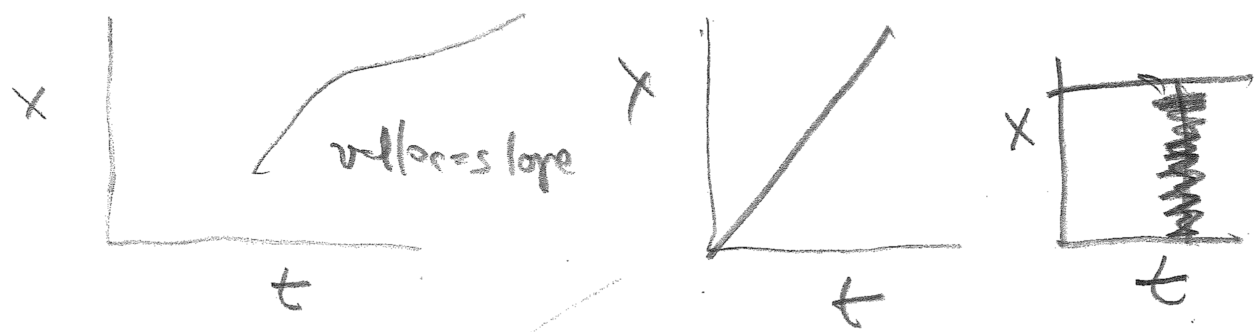
$\Delta L^2 < 0$ time like

$\Delta L^2 = 0$ light like

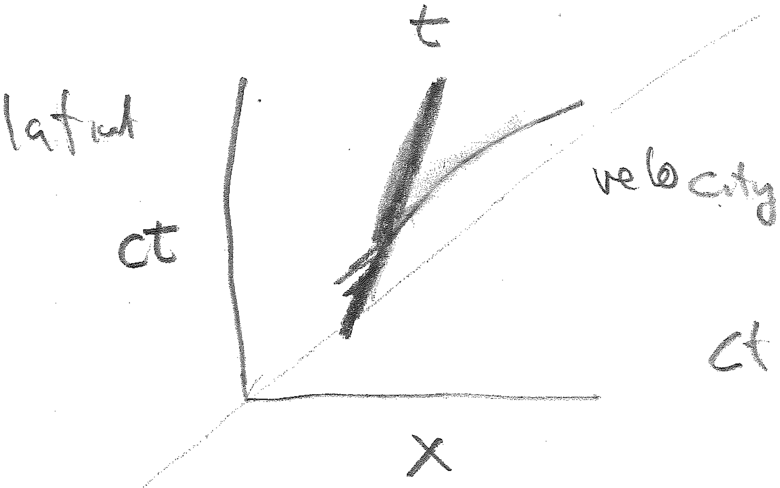
Spacetime diagrams

Plotting motion relativistically

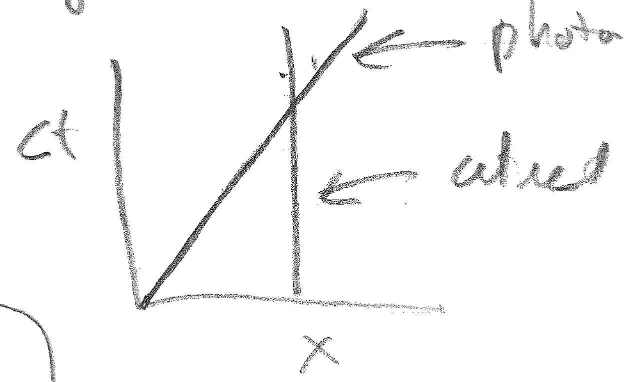
Usually



relativist



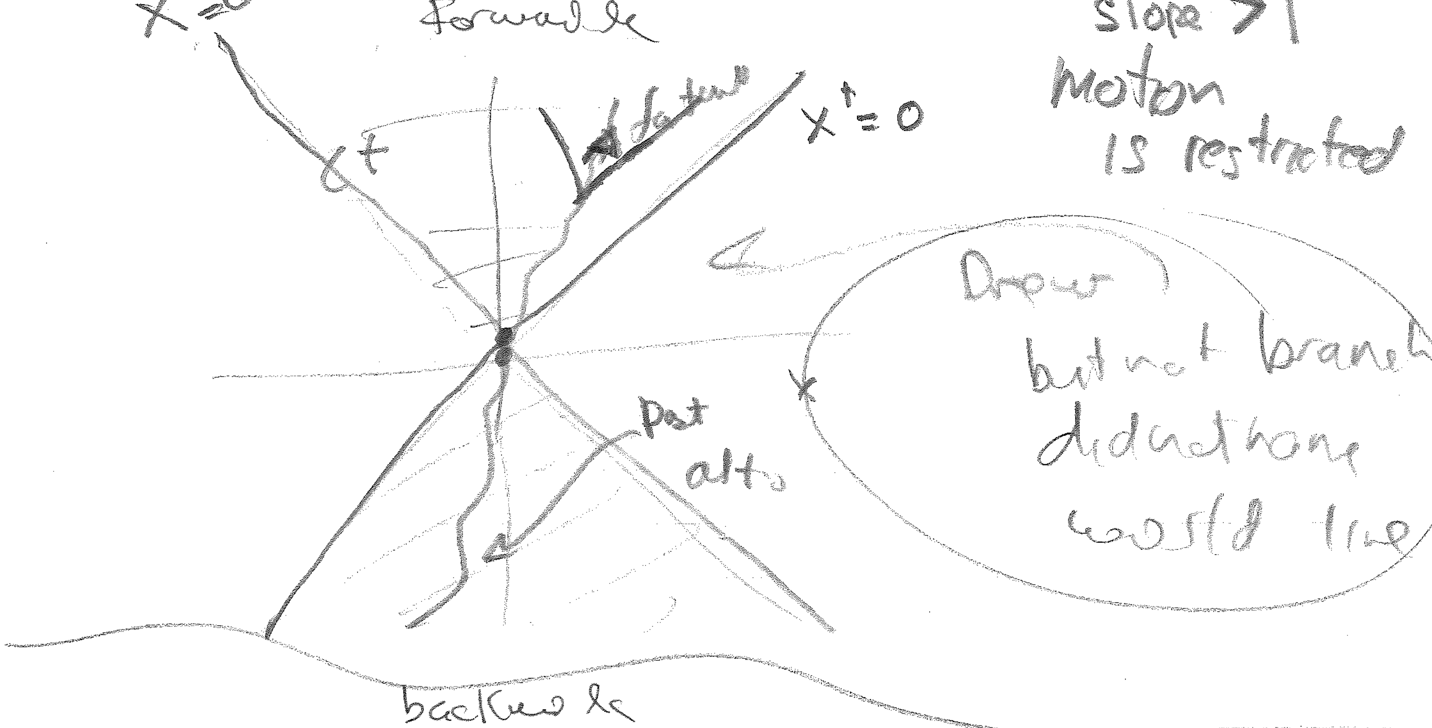
velocity $\propto$ 1/slope



$\boxed{\text{Minkowski diagram}}$

Trajectory or world line

v/c slope > 1
motion is restricted



as you move future changes options narrow

This is 1d. motion

y, z motion

Wedges ~~line~~ cones — hyper cones

Time & space mixed up

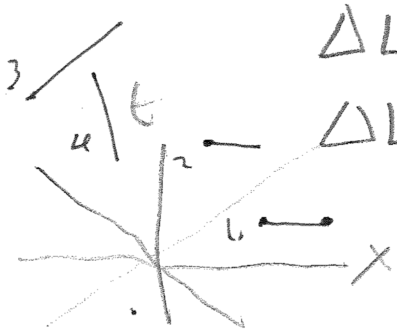
~~xxx~~ What can we do what can't we do

Invariance $\Delta x^2 - c^2 \Delta t^2 = \Delta L^2$

if $\Delta L^2 > 0$ space like (a)

$\Delta L^2 = 0$ light like (b)

$\Delta L^2 < 0$ time like (c)



— Here —

May 28

Last time we discussed Einstein 2 post that are the basis of special theory of relativity

1) Laws of physics same in all IRF
2) Speed of light in vacuum is same in all IRF

This led to time dilation, length contraction and Lorentz transformation and invariant quantities

Two frames S & S'
(x, y, z, t) (x', y', z', t') move with rel. velocity $\vec{v}$

$\vec{X}'_{\parallel} = \gamma(\vec{X}_{\parallel} - \vec{v}t)$ $\parallel$ means $\parallel$ to $\vec{v}$
$\vec{X}'_{\perp} = \vec{X}_{\perp}$ $\perp$ " $\perp$ " "
$t' = \gamma\left(t - \frac{\vec{v} \cdot \vec{X}}{c^2}\right)$

$\vec{v} \cdot \vec{x} = v x_{\parallel}$

Space & time are connected - intertwined

Invariant interval 2 events 1) $(\vec{X}_1, t_1)$
2) $(\vec{X}_2, t_2)$

$\Delta L^2 = \Delta \vec{X}^2 - c^2 \Delta t^2$

$\Delta \vec{X} = \vec{X}_2 - \vec{X}_1$
$\Delta t = t_2 - t_1$

There is something familiar here - |
This ~~should remind u~~ is like rotation
Lengths of vector unchanged when rotating coordinate system

First job today introduce the

Idea of $\boxed{\text{4 vector}}$ We've already seen

One candidate 4-v. $(\vec{x}, t) =$

Space and time are intertwined

We label event by space time 4 ~~vector~~
space time coordinate
$(x, y, z, ct) = x^{\mu}$ ct to make all dimension 4 terms have same
$= (\vec{x}, ct) = (\vec{r}, ct)$

This is a generalization of conventional 3-vectors

The Lorentz transform can be written

as $x'^{\mu} = \Lambda^{\mu}_{\nu}(\vec{v})\, x^{\nu}$ Einstein summation convention

velocity

4vector any set of 4 objects that transforms like space time.

What is another example?

Is there something like velocity or momentum?

~~Need~~ velocity = deriv of space wrt time

need to think of time

Consider two events separated by $d\vec{x}$, dt some system changes by

$$c^2 d\tau^2 = c^2 dt^2 - d\vec{x}^2$$

(proper time interval)^2

proper time interval $\quad d\tau = dt\sqrt{1 - \frac{d\vec{x}^2}{dt^2 c^2}} = dt\sqrt{1 - u^2/c^2} = \frac{dt}{\gamma(u)}$

τ is the proper time of a moving system

it is time as seen in the rest frame of ~~the~~

system. $d\tau = dt$ if $\vec{u} = 0$

The equation $d\tau = dt/\gamma$ is another expression

of time dilation

Now back to velocity

$\vec{u} = \dfrac{d\vec{x}}{dt}$ has ugly transformation

properties both top & bottom are changed

under LT

Try

Proper Velocity $\vec{\eta} = \dfrac{d\vec{x}}{d\tau} = \gamma(u)\dfrac{d\vec{x}}{dt} = \gamma(u)\vec{u}$

This is a 3-vector that transforms as the

3 space components of a 4 vector. Is there a

4th partner. Why not t $\eta^0 = \dfrac{c\,dt}{d\tau} = c\gamma(u)$

$(\eta^0, \vec{\eta})$ form a <u>4 vector transform same as X^μ</u>

This simple idea has amazing consequences

So there are at least 2 - 4 vectors

others - $(c\rho, \vec{J})$ $\left(\dfrac{V}{c}, \vec{A}\right)$ to be shown

There are also 4-tensors of rank 2

Second rank tensor
transforms as $x^\mu x^\nu$, $\tilde{t}^{\mu\sigma} = \Lambda^{\mu}_{\rho}(\vec{v})\Lambda^{\sigma}_{p}(\vec{v})\, t^{\rho p}$

One step at a time need to finish formalism

generalize dot product $\vec{A}\cdot\vec{B}$ invariant

Suppose we have to four vectors A^μ, B^μ

The invariant dot product is

$$-A^0B^0 + \vec{A}\cdot\vec{B} \qquad \begin{pmatrix} -x^{0\,2} + \vec{x}^2 \\ -\eta^{0\,2} + \vec{\eta}^2 \\ -x^0\eta^0 + \vec{\eta}\cdot\vec{x} \end{pmatrix} \text{ examples}$$

Have minus sign in time conponent.

Define $A^\mu = (A^0, \vec{A})$ contravariant 4 vector
$B_\mu = (-B^0, \vec{B})$ covariant 4 vectors
$B_0 = -B^0$ $\quad B_n = B^n$, $n = 1,2,3$

The $A^\mu B_\mu = A_\mu B^\mu = -A^0B^0 + \vec{A}\cdot\vec{B}$
is invariant under LT

A^μ transforms with $\Lambda(\vec{v})$

A_μ transforms with $\Lambda(-\vec{v}) = \Lambda^{-1}(\vec{v})$

example LT in x-dir

$$x'^0 = \gamma\left(x^0 - \frac{v}{c}x'\right)$$

$$x''= \gamma\left(x' - \frac{v}{c}x^0\right)$$

$$-x'^0 = \gamma\left(-x^0 + \frac{v}{c}x'\right)$$

$$x'_0 = \gamma\left(x_0 - \left(-\frac{v}{c}\right)x_1\right) \qquad \text{Inverse LT}$$

We may also write $a_\mu = g_{\mu\nu} a^\nu$

$$g_{\mu\nu} = \begin{pmatrix} -1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix} = g^{\mu\nu}$$

Go back to 4velocity

Define 4 momentum $= m\vec{\eta} = \vec{p}$

$p^\mu = m\eta^\mu$ $\qquad m\eta^0 = ?$

$$\vec{p} = m\gamma\vec{u}$$

$$p^0 = \underline{mc\,\gamma(u)} = E/c$$ dimensions of energy / c

$$E = mc^2\gamma(u)$$

Interpretation: let $\frac{u}{c}$ be $\ll 1$ rest mass energy

$$E = mc^2\left(\frac{1}{\sqrt{1-u^2/c^2}}\right) = mc^2 + \frac{mu^2}{2} - \frac{3}{8}\frac{u^4}{c^2}$$

Non-relativistic kinetic energy, $\underline{E \text{ is energy}}$

expansion good if KE $\ll mc^2$

There should be an invariant related to proper time & proper length

$$E^2 - c^2\vec{p}^{\,2} = \frac{m^2c^4}{1-u^2/c^2} - c^2\frac{m^2u^2}{1-u^2/c^2}$$

$$= \frac{m^2c^4}{1-u^2/c^2}\left(1-\frac{u^2}{c^2}\right) = m^2c^4$$

$$\boxed{E^2 = c^2p^2 + m^2c^4}$$ Einstein relation

Before Conservation of momentum
Conservation of energy in closed systems

Now conservation of 4-momentum

Just as time and space are intertwined

Energy & momentum are connected energy

can be transformed into momentum

How does this work - examples

Conservation of energy + momentum

in collisions reactions decays

$P_i^\mu = \sum_i P_i^\mu$ sum is overall particles

$P_f^\mu = \sum_j P_j^\mu$ " not necessary same particle

$$P_i^\mu = P_f^\mu \quad \text{or} \quad P_i = P_f$$

$$P_i^2 = P_f^2 \quad \text{or}$$

Example 1

Discovery of neutrino $\quad -c^2 P_i^2 = -c^2 P_f^2$

n decays $\quad n \rightarrow p + e^- + \bar{\nu}_e$

$\bar{\nu}_e$ has almost no mass and hardly interacts
not detected

First reaction thought to be $\quad n \rightarrow p + e^-$

$$M_n c^2 = 941.6 \text{ MeV}$$
$$M_p c^2 = 939.3 \text{ MeV}$$

$$\vec{0} = \vec{P}_e + \vec{P}_p , \quad \vec{P}_e = -\vec{P}_p = \vec{P}$$

$$M_n c^2 = \sqrt{c^2P^2 + M_p^2 c^4} + \sqrt{p^2c^2 + M_e^2 c^4}$$

$$cP = 2.23 \text{ MeV}$$

For electron $cp \approx 4.4 \, M_e c^2$

$M_e c^2 = 0.511$ MeV . Reaction occurs releasing $941.6 - 939.3 - .511 = .789$ MeV

Momem conservation - neutron at rest

↑ small OK to expand

Mom $\quad \vec{0} = \vec{P}_p + \vec{P}_e \approx \gamma(v_p)\, m_p \vec{V}_p + \gamma(v_e)\, M_e \vec{V}_e$

energy $\quad M_n c^2 = M_p c^2 + \frac{1}{2} m_p V_p^2 + M_e c^2 + \frac{1}{2} M_e V_e^2$

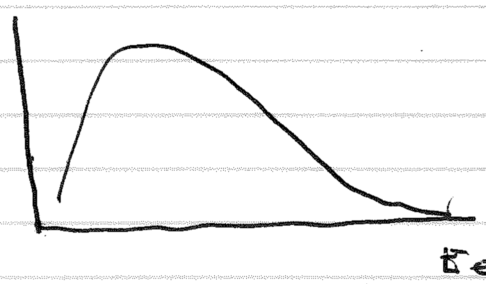
$$V_p = \frac{M_e V_e}{m_p}$$

$$0.789 \text{ MeV} = \frac{1}{2}(939.3 \text{ MeV})\left(\frac{M_e}{M_p}\right)^2 V_e^2 + \frac{1}{2} M_e V_e^2$$

Ve is fixed - one value - researchers

saw

N(e)



V_e

4 momentum conservation violated?

~~Even~~ Pauli said no

there must be another particle

Now ν's are ~~an~~ important subject in Modern physics

Basic thing ν ~~Energy~~ Mass goes into KE of e, P, $\bar{\nu}_e$

One more example discovery of

antiproton $= \bar{p}$ discovery at Bevatron (Berkeley)

$\bar{p}$ has same mass opposite charge as

proton - Dirac 1933 prediction

PR 100 947 (1955)

First Reaction $p + p \to p + p + \bar{p} + p$

minimum energy to make

Let $Mc^2 =$ p mass $= \bar{p}$ mass

~~MIM~~ one p is moving

Initial state $p \longrightarrow$ $\underset{p}{\bullet}$ at rest. How much energy needed

~~on~~ P^μ is conserved

consider $-\hat{c}P_i^\mu P_{i\mu} > s = -\hat{c}P_f^\mu P_{f\mu}$

Lowest energy final state as $4Mc^2$ at rest

$$-\hat{c}P_i^2 = -\hat{c}P_i^\mu P_{i\mu} = (E_L + Mc^2)^2 - P_L^2 c^2$$

$$= E_L^2 + 2E_L Mc^2 + (Mc^2)^2 - (P_L c^2)$$

$$= 2Mc^2(E_L + Mc^2)$$

$$> P_f^\mu P_{f\mu} \;\; = (4Mc^2)^2 = 16M^2c^4$$

$$E_L = \frac{16 M^2 c^4}{2Mc^2} - \; = 7Mc^2 = Mc^2 + T_L$$

$$T_L = 6Mc^2 = 5.6 \text{ BeV}$$

Accelerator highest energy was 6 BeV above threshold. but production cross section too small

Solution use heavy target ^{12}C

$$M_{12}c^2 \approx 12Mc^2$$

$$P_i^2 = (E_L + 12Mc^2)^2 - P_L^2c^2$$

$$= (Mc^2)^2 + 2E_L(12Mc^2) + 144(Mc^2)^2$$

~~P_f~~ final state $\quad {}^{12}C + p + \bar{p}$

$$P_f^2 = (14Mc^2)^2$$

$$E_L = \frac{51(Mc^2)^2}{24Mc^2}$$

$$= \frac{17}{8}Mc^2$$

$T_L = 9/8\ Mc^2 \quad$ production within range

Aside on units $E^2 = p^2c^2 + m^2c^4$

~~Units~~ - Lots of c's - Units

Suppressed units

momenta, energies & masses measured in units of energy, velocities to rate of c

$$mc^2 \rightarrow m$$
$$pc \rightarrow p$$
$$E \rightarrow E$$
$$v/c \rightarrow v/c$$

MeV
GeV
$m_p = .938$ GeV
$= 938$ MeV

Then see lots

$E^2 = p^2 + m^2$ instead of $E^2 = p^2c^2 + m^2c^4$

<u>Relativistic dynamics</u> how things move

Suppose Constant force how long does it take a particle of mass m starting at rest to reach $u = c/2$

$$\vec{F} = \frac{d\vec{p}}{dt}$$ is correct relativistic THE LAW

<u>IF</u> $$\vec{p} = m\gamma(u)\,\vec{u} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}}$$

with a constant force in a given direction particle accelerates in same direction

$$F = \frac{d}{dt}\frac{mu}{\sqrt{1-u^2/c^2}}$$

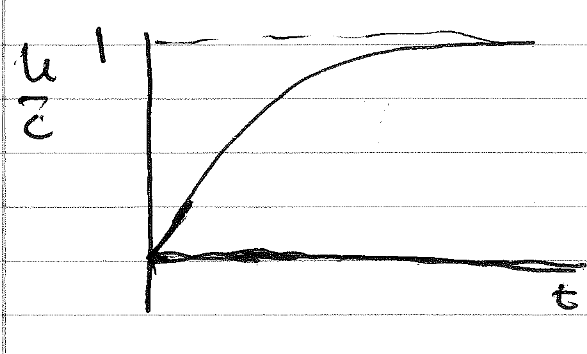
Relativistic effect

$$\frac{F}{m}t = \frac{u}{\sqrt{1-u^2/c^2}} \quad \text{(I)}$$ algebraic equation

can see main features

small t $\quad u = F/m\, t = at \qquad x = \frac{at^2}{2}$

$t \to \infty$ LHS $\to \infty$, RHS $\to \infty$, $u/c \to 1$



Can Solve (5) $\quad u/c = \beta$

$$\frac{LHS}{c} = \frac{Fct}{mc^2} \qquad \frac{RHS}{c} = \frac{\beta}{\sqrt{1-\beta^2}}$$

$[Fct]$ = energy

$[mc^2]$ = "

$$\beta = \frac{\frac{Fct}{mc^2}}{\sqrt{1+\left(\frac{Fct}{mc^2}\right)^2}}$$

To get $x(t) = \int_0^t u(t')\,dt'$ integrate

$$x = \frac{mc^3}{F}\left[\sqrt{1+\left(\frac{Ft}{mc}\right)^2} - 1\right]$$

Low t $\quad x = \frac{mc^3}{F}\left(1 + \frac{1}{2}\left(\frac{Ft}{mc}\right)^2 - 1\right)$

$$= \frac{1}{2}\frac{F}{m}t^2$$

Large t $\quad x = \frac{mc^3}{F} \cdot \frac{Ft}{mc} = ct$

Transforming $\vec{E}$ & $\vec{B}$

We have seen that X^μ, p^μ are fourvectors

& know how to write these in different frames

<u>What about $\vec{E}$ & $\vec{B}$ – are they part of</u>

4vector – no!

Remind vectors tensor

$\vec{F} = \frac{d\vec{p}}{dt}$ $\quad \vec{p} = m\gamma\vec{u}$ $\quad$ Fact

If $\vec{E}$ electric field on charge q, $\vec{F} = q\vec{E}$

$q\vec{E} = \frac{d\vec{p}}{dt}$

ratio of components of two different 4-vectors p^μ, x^μ

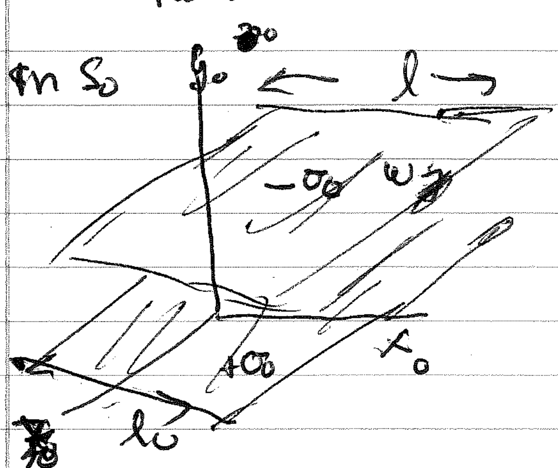
We'll see this expression is component of rank-2- Lorentz tensor

Goal – <u>how do $\vec{E}$ & $\vec{B}$ transform?</u>

Work this out with simple example

Suppose have uniform $\vec{E}$ in large ‖ plate capacitor

In S_0



$\vec{E}_0 = \frac{\sigma_0}{\epsilon_0}$ inside the plates

Let's examine this same capacitor ~~in~~ ~~system~~ system in frame

S moves right with speed v_0, What's constant? Charge

in S - system moves leftward

but $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{y}$

σ is different because lengths are different

the length l is contracted by

$\gamma_0 = \frac{1}{\sqrt{1-v_0^2/c^2}}$

Any area on the plate is decreased by γ, Charge is conserved charge is a Lorentz scalar

charge/unit area increased

$\sigma = \frac{\sigma_0}{\sqrt{1-v_0^2/c^2}} = \gamma\sigma_0$

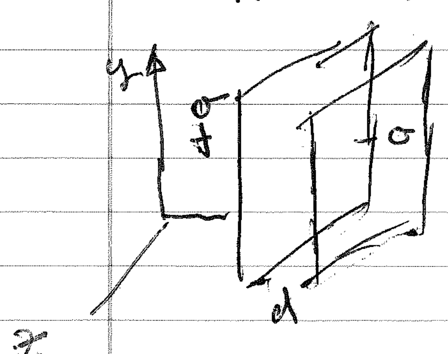
Thus $\vec{E}_y$ is increased more generally

$\hat{y} \perp \vec{v}$ direction of motion

$\vec{E}^{\perp} = \gamma_0 \vec{E}_0^{\perp}$

how about components $\parallel$ to $\vec{v}$

Suppose system looked



The separation d is contracted charge density unchanged

Thus $E^{\parallel} = E_0^{\parallel}$

Summary

$$\vec{E}^{\perp} = \gamma_0 \vec{E}_0^{\perp}, \quad \vec{E}^{\parallel} = \vec{E}_0^{\parallel}$$

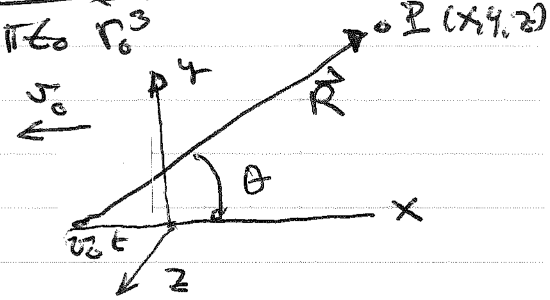
$\boxed{\text{The transverse component of the electric field is smaller in the frame with charges at rest than in any other frame}}$

Next

$\vec{E}$ field of moving point charge using these rules

In frame S_0 point charge at rest. What is $\vec{E}$ in S moving to right in which charges moves to left with v_0

In S_0, $\vec{E}_0 = \dfrac{q\,\vec{r}_0}{4\pi\epsilon_0 r_0^3}$



$$E_{0x} = \frac{q\,x_0}{4\pi\epsilon_0 (x_0^2+y_0^2+z_0^2)^{3/2}}$$

$$E_{0y} = \frac{q\,y_0}{4\pi\epsilon_0 (x_0^2+y_0^2+z_0^2)^{3/2}}$$

$$E_{0z} = \frac{q\,z_0}{4\pi\epsilon_0 (x_0^2+y_0^2+z_0^2)^{3/2}}$$

$\vec{v}_0$ in x direction y & z are $\perp$ to x

$$E_y = \gamma_0 E_{0y} \qquad \gamma_0 = \frac{1}{\sqrt{1-v_0^2}}$$
$$E_z = \gamma_0 E_{0z}$$

So E_y & E_z each get a γ factor

~~plus~~ but we need to express x_0, y_0, z_0 in S_0

in terms of x, y, z in S

$$x_0 = \gamma_0(x + v_0 t) = \gamma_0 R_x$$

x component of vector from q to P

$$y_0 = y = R_y, \quad z_0 = z = R_z$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q\,\gamma_0 \vec{R}}{\left[\gamma_0^2 R_x^2 + R_y^2 + R_z^2\right]^{3/2}}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q\,\gamma_0 \vec{R}}{\left[\gamma_0^2 R^2 \cos^2\theta + R^2 \sin^2\theta\right]^{3/2}}$$

Electric field of a charge that has been

moving ~~with~~ uniformly is at any instant

of time directed radially outward.

from instantaneous position of charge

If $v_0/c \ll 1$ get Coulomb's Law

This result (with some algebra)

is same as eq 10.75 from LW potentials

In the frame S there is a current $\vec{J}(\vec{r},t) = \rho \vec{v}$

$$= q\delta(\vec{r} - \vec{v}_0 t)\, \vec{v}_0$$

There fore there is a magnetic field

The magnetic field calculated in Chapt 10 from L-W

$$\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$$

$\vec{B}$ in the frame S_0 was 0

But it is not zero in S

This is an example of the general frame

If frames related by $\vec{v}$ in x-direction (opposite to S_0, S)

S $\bar{S}$

$$\bar{E}_x = E_x \qquad \bar{E}_y = \gamma(E_y - vB_z) \qquad \bar{E}_z = \gamma(E_z + vB_y)$$

$$\bar{B}_x = B_x \qquad \bar{B}_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right) \qquad \bar{B}_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

More general $\vec{v}$

$$\vec{\bar{E}}_{\parallel} = \vec{E}_{\parallel} \qquad \vec{\bar{E}}_{\perp} = \frac{\vec{E}_{\perp} + \vec{v} \times \vec{B}}{\sqrt{1 - v^2/c^2}}$$

$$\vec{\bar{B}}_{\parallel} = \vec{B}_{\parallel} \qquad \vec{\bar{B}}_{\perp} = \frac{B_{\perp} - \vec{v} \times \vec{E}/c^2}{\sqrt{1 - v^2/c^2}}$$

This is NOT a Lorentz trans

$\vec{E}$, $\vec{B}$ not part of 4-vectors

Covariance of Electrodynamics

Postulate 1 of relativity – the eqns of physics have same form in all IRFs. This is called covariance. Covariance is automatically true if they can be written in tensor form

example: $V_1^{\mu} = V_2^{\mu}$, $T_1^{\mu\nu} = T_2^{\mu\nu}$

last time $P^{\mu}_{final} = P^{\mu}_{initial}$ ← "manifestly covariant"

Our goal in remainder of course – put equations of electrodynamics & Maxwell in manifestly covariant form

Start with Newton's second

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \text{where} \quad \vec{p} = m\gamma(u)\vec{u}$$

True relativistically – want to write in terms of 4-vectors & 4-tensors.

This is not the case now ~ $\vec{F}$ is a ratio of components of 4 vectors

Let's define (in analogy with η^{μ}) a four vector

$$\frac{dp^\mu}{d\tau} = K^\mu$$ (Minkowski force)

It is a $\underline{4}$ $\underline{\text{vector}}$

Manifestly covariant eq of motion

Try to understand K^μ better $\quad d\tau = \frac{1}{\gamma} dt$

$$\vec{K} = \frac{d\vec{p}}{d\tau} = \frac{d\vec{p}/dt}{d\tau/dt} = \frac{\vec{F}}{\sqrt{1-v^2/c^2}}$$

$$K^0 = \frac{1}{c}\frac{dE}{d\tau} = \frac{dE/dt}{c\, d\tau/dt} = \frac{\vec{F}\cdot\vec{v}/c}{\sqrt{1-v^2/c^2}}$$ ← work energy theorem

N's 2nd has complicated ~~fr~~ transform properties, but K^μ is a fourvector

<u>Let's do the rest of E&M in covariant</u>

<u>form</u> start with Lorentz force Law

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\vec{v}\times\vec{B}$$

P_i and dt each transform as a component of 4vector — suggest LHS & RHS are parts of 2nd rank tensor

Let's use K^μ to get started, take 3 vector part first

1. $\vec{K} = \dfrac{q\vec{E} + q\vec{v}\times\vec{B}}{\sqrt{1-v^2/c^2}} = q\eta^0 \dfrac{\vec{E}}{c} + q\vec{\eta}\times\vec{B}$ $\quad \gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$

↗ looks like $\eta^\mu = dx^\mu/d\tau$ enters

or $K^i = q\eta^0 \dfrac{E^i}{c} + q\,\epsilon_{ijk}\,\eta^j B^k$

2. $K^0 = \gamma\, q\vec{E}\cdot\vec{v} = \dfrac{\gamma}{c}\, q\,\vec{E}\cdot\vec{v} = q\,\vec{\eta}\cdot\vec{E}$

Minkowski combined $\vec{E}$ & $\vec{B}$ into an antisymm Lorentz tensor - electromagnetic field strength tensor $F^{\mu\nu}$ antisymmetric

$F^{ij} \equiv \epsilon_{ijk} B^k$ $\qquad B^k = B_x, B_y, B_z$

$F^{0i} = -F^{i0} = E^i/c$ $\qquad$ • See $F^{\mu\nu}$ on next page

$F^{00} = 0$

Then 1 & 2 can be combined into

$$\boxed{K^\mu = q\,\eta_\nu F^{\mu\nu}}$$

new idea
~~dot~~ product
between a 4 vector
and 4 tensor rank 2
is 4 vector

HW Example $X_\nu(X^\mu X^\nu) = X^\mu(X_\nu X^\nu)$ ← scalar
transforms as a 4-vector

$$K^0 = \frac{q\,\vec{E}\cdot\vec{u}/c}{\sqrt{1-u^2/c^2}} = q\vec{\eta}\cdot\frac{\vec{E}}{c}$$

Minkowski idea: E, B combine in a tensor. Elect. magnetic field tensor

$F^{\mu\nu}$ $\qquad F^{ij} = \epsilon_{ijk} B^k$

$$F^{0i} = -F^{i0} = E^i/c$$

$$F^{00} = 0$$

$F^{\mu\nu}$ antisymm. Field tensor. Not proved yet

$$F^{\mu\nu} = \begin{array}{c|cccc} \mu \backslash \nu & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & E_x/c & E_y/c & E_z/c \\ 1 & -E_x/c & 0 & B_z & -B_y \\ 2 & -E_y/c & -B_z & 0 & B_x \\ 3 & -E_z/c & B_y & -B_x & 0 \end{array}$$

claim $\boxed{K^\mu = q\,\eta_\nu F^{\mu\nu}} = \frac{dp^\mu}{d\tau}$

space

$$K^i = q\left[\eta_0 F^{i0} + \eta_j F^{ij}\right]$$

$$= \qquad [-\eta^0(-E^i) + \eta_j\, \epsilon_{ijk} B^k] \text{ OK}$$