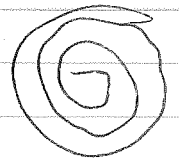
Continuation of previous Lecture

This solution assumes motion



For this to be true

must have decrease in R in one period (T) small compared to R

$$\frac{\Delta R}{R} = \frac{\dot{R}\,T}{R} = \frac{\dot{R}}{c}(2\pi\alpha) \qquad T = \frac{2\pi a_0}{v}, \text{ Bohr atom}$$

$$v = \alpha c$$

$$\alpha = \frac{1}{137}$$

Determine $\frac{\dot{R}}{c}$

$$\dot{R} = -\frac{\mu_0}{12\pi^2}\frac{1}{\epsilon_0 c}\frac{q^4}{m^2 R^2}$$

$$q^2 = 4\pi\epsilon_0 \hbar c \alpha$$

$$\dot{R} = \frac{4}{3}\frac{\cancel{\mu_0}\cancel{\epsilon_0}}{c}\frac{\hbar^2 \cancel{c^2}}{m^2 R^2}\alpha^2$$

$$= \left(\frac{4}{3}\frac{\hbar^2 c^2}{m^2 c^4}\frac{\alpha^2}{R^2}\right)c$$

( ) is adimensionless number

Use R = Bohr radius = $.53 \times 10^{-10}$ m

$$(\ ) = \frac{4}{3}\frac{(197\text{ MeV fm})^2}{(.511\text{ MeV})^2}\frac{1}{.53^2\times 10^{-20}\text{ m}^2} \approx 10^{-4}$$

Thus $\frac{\Delta R}{R} = 10^{-4} \frac{2\pi}{137} = 5 \times 10^{-6}$

Very small

May 14

and generalization of

Summary of Radiation Physics

This is a more general version of Sect 11.1.4 of Griffiths Radiation from an arbitrary source

1 Calculating the Fields

$\rho(\vec{r}', t)$
$\vec{J}(\vec{r}', t)$
in a finite region of space

Maxwell's equations can be expressed (in the Lorentz gauge) as

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A}(\vec{r},t) = -\mu_0 \vec{J}(\vec{r},t), \qquad \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)V(\vec{r},t) = -\frac{\rho(\vec{r},t)}{\epsilon_0}. \quad (1)$$

The Lorentz gauge is defined by

$$\vec{\nabla}\cdot\vec{A}(\vec{r},t) + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0. \quad (2)$$

Lorentz gauge

The derivations of these equations were given in class and are in Griffiths.

Conservation of current:

$$\vec{\nabla}\cdot\vec{J}(\vec{r},t) + \frac{\partial \rho(\vec{r},t)}{\partial t} = 0 \quad (6)$$

Statement of charge conservation

One may note that Eqs. (6) and (2) have a similar appearance, with $\vec{A}$ playing the role of $\vec{J}$ and V/c^2 playing the role of ρ. This is no accident, as we shall see when we study four vectors in relativity.

It is useful to work in frequency space. The Fourier transformations are defined as (FT)

$$\vec{A}(\vec{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, \vec{A}(\vec{r},\omega)e^{-i\omega t}, \qquad \vec{J}(\vec{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \vec{J}(\vec{r},\omega)e^{-i\omega t}. \tag{3}$$

Similar equations hold for $\Phi(\vec{r},t)$ and $\rho(\vec{r},t)$. Then the time derivative can be replaced by a factor: $\frac{\partial}{\partial t} \to -i\omega$ and we can find equations that determine $\vec{A}(\vec{r},\omega)$ and $\Phi(\vec{r},\omega)$:

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\vec{A}(\vec{r},\omega) = -\mu_0\vec{J}(\vec{r},\omega) \qquad \left(\nabla^2 + \frac{\omega^2}{c^2}\right)\Phi(\vec{r},\omega) = -\frac{\rho(\vec{r},\omega)}{\epsilon_0}. \tag{4}$$

We can also FT eq 6

$$\vec{\nabla}\cdot\vec{J}(\vec{r},\omega) - i\omega\rho(\vec{r},\omega) = 0$$

This is the same as having $\vec{J}(\vec{r},t)$ and $\rho(\vec{r},t)$ of the form $\vec{J}(\vec{r},\omega) = \vec{J}(\vec{r})e^{-i\omega t}$ and $\rho(\vec{r},\omega) = \rho(\vec{r})e^{-i\omega t}$. These last two forms are obtained in quantum mechanics for which, in a radiative transition from an initial state i to a final state f: the photon energy $\hbar\omega = E_i - E_f$, and $\langle f \mid \vec{J}(\vec{r}) \mid i\rangle \to \vec{J}(\vec{r})$. These last two forms also occur is situations in which the time dependence of the source is represented by one frequency, and taking the real part is implicitly understood.

We've already written expressions for $\vec{A}, V$

$$\begin{pmatrix} V(\vec{r},t) \\ \vec{A}(\vec{r},t) \end{pmatrix} = \frac{1}{4\pi} \int \frac{d^3r'\,dt'}{|\vec{r}-\vec{r}'|} \begin{pmatrix} \rho(\vec{r}',t')/\epsilon_0 \\ \mu_0 \vec{J}(\vec{r}',t') \end{pmatrix} \delta\left(t-t'-\frac{|\vec{r}-\vec{r}'|}{c}\right)$$

Inv. FT of this gives $V(\vec{r},\omega)$, $\vec{A}(\vec{r},\omega)$

To solve [redacted] do inverse FT

multiply above by $\int_{-\infty}^{\infty} dt\, e^{i\omega t}$.

[This is an inverse FT. The LHS goes to $V(\vec{r},\omega)$, $\vec{A}(\vec{r},\omega)$]

Use $\int dt\, e^{i\omega t}\, \delta\left(t - t' - \frac{|\vec{r}-\vec{r}'|}{c}\right)$

$$= e^{i\left(t' + \frac{|\vec{r}-\vec{r}'|}{c}\right)\omega}$$

Then the integrals [redacted]

$$\int dt'\, e^{i\omega(t' + |\vec{r}-\vec{r}'|/c)} \begin{bmatrix} \rho(\vec{r}',t')/\epsilon_0 \\ \vec{J}(\vec{r}',t')\,\mu_0 \end{bmatrix}$$

appear. These are

$$= \begin{pmatrix} \rho(\vec{r}',\omega)/\epsilon_0 \\ \vec{J}(\vec{r}',\omega)\,\mu_0 \end{pmatrix}$$

It is worthwhile to examine $V(\vec{r},\omega)$

First we are concerned with radiation from a compact (finite-sized) source

so $r \gg r'$ for all r' where sources $\rho, \vec{J}$ do not vanish

$$|\vec{r}-\vec{r}'| \Rightarrow r - \hat{r}\cdot\vec{r}'$$

$$V(\vec{r},\omega) \approx \frac{e^{ikr}}{4\pi r}\int d^3r'\, e^{-i\vec{k}\cdot\vec{r}'}\,\rho(\vec{r}',\omega)/\epsilon_0$$

$$\vec{k} \equiv k\hat{r}$$

→ Another FT 3 dimensional spatial FT

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}; \quad kr' = \frac{2\pi r'}{\lambda}$$

Long wavelength approx $2\pi r'/\lambda \ll 1$

Expand exponential

$$e^{-i\vec{k}\cdot\vec{r}'} \approx 1 - i\vec{k}\cdot\vec{r}' + \cdots$$

$$V(\vec{r},\omega) = \frac{e^{ikr}}{4\pi r}\int d^3r'\left(1 - i\vec{k}\cdot\vec{r}'\right)\rho(\vec{r}',\omega)/\epsilon_0$$

$$\boxed{V(\vec{r},t) = \int \frac{e^{-i\omega t}}{2\pi}\,d\omega} = \frac{1}{\epsilon_0 4\pi r}\left[\int d^3r'\,\rho(\vec{r}',\omega) - i\vec{k}\cdot\vec{p}(\omega)\right]$$

Fourier Transform $\vec{p}(\omega) = \int d^3r'\,\vec{r}'\,\rho(\vec{r}',\omega)$

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\frac{1}{r}\left[\underbrace{\int d^3r'\,\rho(\vec{r}',t_0)}_{\text{total charge}} - i\int\frac{d\omega}{2\pi}e^{-i\omega t}\frac{\omega}{c}\vec{p}(\omega)\right]$$

$t_0 = t - r/c$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{r}Q(t_0) + \frac{1}{4\pi\epsilon_0}\frac{\partial}{\partial t}\vec{p}(t)\frac{1}{c}$$

But charge conserved $Q(t_0) = Q$

$$\vec{E} = -\vec{\nabla}V + \frac{\partial\vec{A}}{\partial t}$$

Take $-\vec{\nabla}\frac{Q}{r}$ to get its contribution to $\vec{E}$

gives terms proportional $1/r^2$

Charge Monopole does <u>not</u> radiate

The dipole contribution to V is

proportional to $\frac{d\vec{p}}{dt}$

But we do not really need V to get $\vec{E}$ & $\vec{B}$ in radiation zone

Can compute $\vec{A}$, $\vec{B} = \vec{\nabla} \times \vec{A}$ and get $\vec{E}$ from $\vec{\nabla} \times \vec{B} = +\frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$

General treatment of

3 Radiated Power

We are concerned with a localized system of charges and currents, so that we can speak of positions far away from the charge and current distribution. For such positions $r \gg r'$ for **any** points in the integration over $\vec{J}(\vec{r}\,',\omega)$ for which there is a non-vanishing value of $\vec{J}(\vec{r}\,',\omega)$. Such positions are defined to be in the radiation zone and we find that

$$\vec{A}(\vec{r},\omega) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r'\,\vec{J}(\vec{r}\,',\omega)e^{-i\vec{k}\cdot\vec{r}\,'}, \tag{8}$$

with $\vec{k} \equiv k\hat{r}$. Note that this expression arises from taking $r \gg r'$

The vector field $\vec{A}(\vec{r},\omega)$ is proportional to the spatial Fourier transform of $\vec{J}(\vec{r}\,',\omega)$:

$$\vec{J}(\vec{k},\omega) \equiv \int d^3r'\,\vec{J}(\vec{r}\,',\omega)e^{-i\vec{k}\cdot\vec{r}\,'}, \tag{10}$$

so that

$$\vec{A}(\vec{r},\omega) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{J}(\vec{k},\omega). \tag{11}$$

Often The [redacted] useful observable is the angular distribution of time-averaged radiated power. This is to be obtained from the time-averaged Poynting vector $\frac{1}{2}\vec{E}(\vec{r},\omega)\times\vec{H}^*(\vec{r},\omega)$. So we need $\vec{E}$ and $\vec{H}$. We have $\vec{B} = \mu_0\vec{H} = \vec{\nabla}\times\vec{A}$. We are interested in keeping only those terms that survive in the radiation zone. These are the ones that fall as r^{-1}, any terms falling faster with r do not contribute to the radiated power. Since the vector potential has a e^{ikr}/r behavior, the only terms that survive taking the curl are those in which the gradient acts on the exponential. Thus for positions in the radiation zone, and for calculations of the power it is often expedient and correct to make the replacement $\vec{\nabla} \to ik\hat{r}$. Thus we obtain $\vec{B}(\vec{r},\omega)$ for positions in the radiation zone:

$$\vec{B}(\vec{r},\omega) = ik\frac{e^{ikr}}{4\pi r}\hat{r}\times\vec{J}(\vec{r},\omega)\,\mu_0 \tag{12}$$

The next step is to get $\vec{E}(\vec{r},\omega)$. One may compute $\vec{V}$ for positions in the radiation zone and determine $\vec{E}$ from its definition in terms of $\vec{V}$ and $\vec{A}$. However, it's easier to use the Maxwell equation $\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$ or

$$\vec{\nabla}\times\vec{B}(\vec{r},\omega) = -i\omega\frac{1}{c^2}\vec{E}(\vec{r},\omega). \tag{13}$$

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The net result is

$$\vec{A}(\vec{r},\omega) = \frac{\mu_0}{4\pi}\int d^3r'\,\vec{J}(\vec{r}\,',\omega)\frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}, \quad k = \frac{\omega}{c}$$

$$V(\vec{r},\omega) = \frac{1}{4\pi\epsilon_0}\int d^3r'\,\rho(\vec{r}\,',\omega)\frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}. \qquad (5)$$

We need only $\vec{A}$ (not V) to compute the fields $\vec{E},\vec{B}$ and the radiated power.

Why?

This formalism is very general but it is not well adapted to the situation of harmonic motion:

$$\vec{r}_Q = \vec{r}_0 \cos\omega_0 t$$

Here $\rho(\vec{r},t) = q\,\delta(\vec{r} - \vec{r}_0\cos\omega_0 t)$

$$\vec{J}(\vec{r},t) = q\,\vec{V}_Q(t)\,\delta(\vec{r} - r_0\cos\omega_0 t)$$

$$\vec{V}_Q(t) = -\omega\vec{r}_0\sin\omega_0 t$$

No overall $e^{-i\omega t}$ dependence

Fourier transform to $\rho(\vec{r},\omega)$, $\vec{J}(\vec{r},\omega)$ difficult to use

This and (11) leads to

$$
\begin{aligned}
\vec{E}(\vec{r},\omega) &= \\
&= \qquad\qquad = -c\hat{r}\times\vec{B}(\vec{r},\omega) \\
\vec{E}(\vec{r},\omega) &= \frac{-ik}{4\pi}\mu_0\,\frac{e^{ikr}}{r}\hat{r}\times\left(\hat{r}\times\vec{J}(\vec{k},\omega)\right).
\end{aligned}
$$

We used $k=\omega/c,\ c=1/\sqrt{\mu_0\epsilon_0}$ and $Z_0\equiv\sqrt{\frac{\mu_0}{\epsilon_0}}$. The direction of $\vec{E}(\vec{r},\omega)$ is the polarization of the radiation.

The time-averaged power radiated into an area $dA=r^2d\Omega$ located at a position $\vec{r}$ in the radiation zone is given by

$$dP = r^2 d\Omega\,\hat{r}\cdot\vec{S} \tag{15}$$

so that

$$
\begin{aligned}
\frac{dP}{d\Omega} &= r^2\frac{\hat{r}}{2}\cdot\left(\vec{E}(\vec{r},\omega)\times\vec{H}^*(\vec{r},\omega)\right) \\
&= \frac{Z_0}{32\pi^2}k^2\mid\hat{r}\times\vec{J}(\vec{k},\omega)\mid^2 \qquad (16)\\
&= \frac{Z_0}{32\pi^2}k^2\mid\hat{r}\times(\hat{r}\times\vec{J}(\vec{k},\omega))\mid^2,
\end{aligned}
$$

in which vector $\hat{r}\times(\hat{r}\times\vec{J}(\vec{k},\omega))$ gives the direction of polarization.

We may evaluate the vector cross products above to obtain a more specific form of the angular distribution:

$$\boxed{\frac{dP}{d\Omega}=\frac{Z_0k^2}{32\pi^2}\left(\mid\vec{J}(\vec{k},\omega)\mid^2-\mid\hat{r}\cdot\vec{J}(\vec{k},\omega)\mid^2\right).} \tag{17}$$

Completely general timeaverage

4 Long Wavelength Approximation

The wave number $k=\frac{2\pi}{\lambda}$, where λ is the wavelength. In the approximation, $\lambda\gg r'$ for all of the values of r' appearing (10) for $\vec{J}(\vec{k},\omega)$. Using this means that we may say

$$\vec{J}(\vec{k},\omega)\approx\int d^3r'\,\vec{J}(\vec{r},\omega).$$

Localized charges

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recover main dipole features

$$\frac{dP}{d\Omega} \propto \omega^4 p^2 \sin^2\theta$$

$|J|^2 \propto \omega^2 p^2$

along with eq(17) factor of $k^2 = \omega^2/c^2$

gives ω^4. The rest of eq(17):

$$|\vec{J}|^2 - |\hat{r}\cdot\vec{J}|^2$$

& $\cos\theta = \hat{r}\cdot\hat{p}$ th

$$|J|^2 - |\hat{r}\cdot\vec{J}|^2 = |\vec{p}|^2(1-\cos^2\theta)$$

$$= |p|^2 \sin^2\theta$$

Show formalism agrees with previous in LWL

The longwavelength approx is el dipole

Now have $\int d^3r' \vec{J}(\vec{r}', \omega)$ why el dipole?

Well it is a vector. Can use current conservation

$$\vec{\nabla} \cdot \vec{J}(\vec{r}, \omega) = i\omega \rho(\vec{r}, \omega)$$

mult by r_i and integrate

$$\int d^3r \; r_i \vec{\nabla} \cdot \vec{J}(\vec{r}, \omega) = i\omega \int d^3r \; r_i \rho(\vec{r}, \omega)$$

$$= i\omega P_i(\omega)$$

LHS looks like integrate by parts

$$\int d^3r \; r_i \frac{\partial}{\partial r_n} J_n = \int d^3r \frac{\partial}{\partial r_n}\left[r_i J_n - \delta_{in} J_i\right]$$

$$= -\int d^3r \, J_i + \int_{-\infty}^{\infty} dr_n \frac{\partial}{\partial r_n} r_i J_n \to 0$$

so $\boxed{\int d^3r \vec{J}(\vec{r}, \omega) = -i\omega \vec{P}(\omega)}$

Dipole appears

Let's go beyond LWL in Eq(8)

$$\vec{J}(\vec{r},\omega) = \int d^3r' \, \vec{J}(\vec{r}',\omega)\left(1 - i\vec{k}\cdot\vec{r}'\right)$$

The contribution to $\vec{A}$ of term

is $$\vec{A}(\vec{r},\omega) = \frac{e^{ikr}\mu_0}{4\pi r}(-ik)\cdot\int d^3r' \, \hat{r}\cdot\vec{r}'\left(\vec{J}(\vec{r}',\omega)\right)$$

Study vector

$\hat{r}\cdot\vec{r}'\,\vec{J}(\vec{r}',\omega)$ has 3 vectors

vector identity

$$\hat{r}\cdot\vec{r}'\,J_k = r'_i\,J_k\,\hat{r}_i = \frac{r'_i}{r}\left[\frac{1}{2}\left(J_k\hat{r}_i + \hat{r}_k J_i\right) + \frac{1}{2}\left(J_k\hat{r}_i - \hat{r}_k J_i\right)\right]$$

$$\hat{r}\cdot\vec{r}'\,J_k = \frac{1}{2}\left(\vec{r}'\cdot\hat{r}\,J_k + (\vec{r}'\cdot\vec{J})\,\hat{r}_k\right) + \frac{1}{2}(\vec{r}'\times\vec{J})\times\hat{r}$$

Focus on second term

$$\int d^3r' \, \frac{1}{2}(\vec{r}'\times\vec{J}(\vec{r}')) = \vec{m} \text{ magnetic moment}$$

$$\vec{A}_{mag} = \frac{ik\mu_0}{4\pi}(\hat{r}\times\vec{m})\frac{e^{ikr}}{r}$$

$\hat{r}$ unitvector in $\vec{r}$ direction (observer)

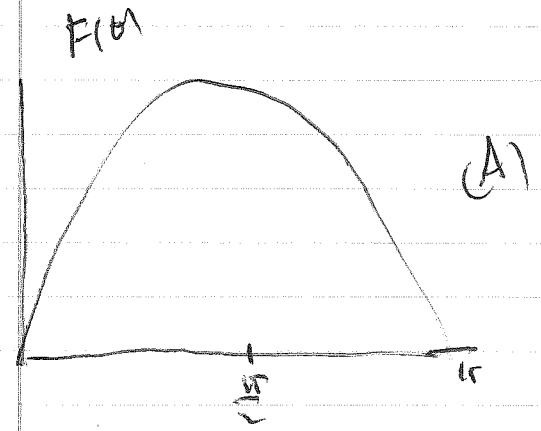
Compare e dipole with m dipole

$$\frac{A_{mag}}{A_{elp}} \sim \frac{k_2 m}{\omega p} \sim \frac{\int d^3r \frac{2\pi r}{\lambda} J}{\int d^3r\, J}$$

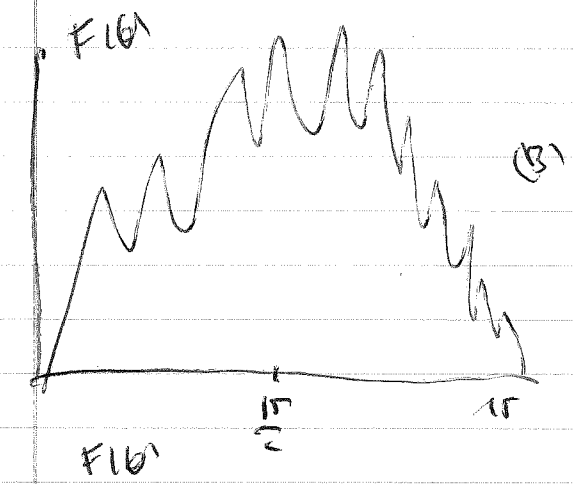
in LWA A_{edp} wins big

Example 3 plots of $F(\theta)$ vs θ

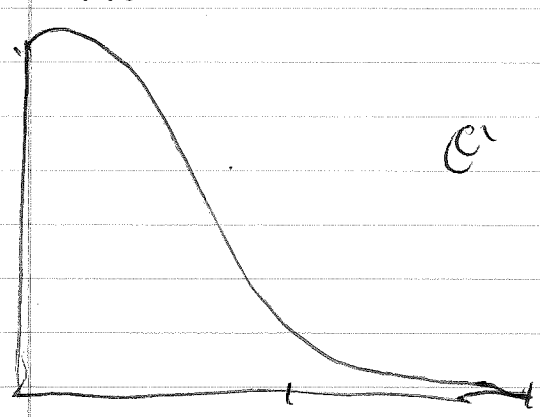
a)



b)



c)



which can't be $\frac{dP}{d\Omega}$

which can't be electric dipole

Practise problem In hydrogen atom (transition) dipole moment written as

$$\vec{p} = -eR(\hat{x} - i\hat{y})e^{-i\omega t}$$

determine $\frac{dP}{d\Omega}$ answer $\propto e^2R^2\omega^4(1+\cos^2\theta)$

Radiation reaction

accelerating charged particle of mass m charge q radiates energy

particle must slow down.

Deceleration implies a force

This is called $\vec{F}_{rad}$ (radiation reaction

Larmor

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} = \frac{\mu_0 q^2 \dot{v}^2}{6\pi c}$$

There must be an external force $\vec{F}_{ext}$ causing acceleration

Energy conservation says that

$$\vec{F}_{rad} \cdot \vec{v} = -\frac{\mu_0 q^2 \dot{v}^2}{6\pi c}$$

rate of doing work

Formula is not good

includes effects of fields due to acceleration

not other terms - fields E, B $\propto 1/r^2$ do not take energy to infinity

Can take a time average

so on average they cancell

$$\overline{\vec{F}_{rad} \cdot \vec{v}} = -\frac{\mu_0 q^2 \overline{\dot{v}^2}}{6\pi c}$$

Want to simplify

$$\dot{v}^2 = \frac{d}{dt}(\vec{v} \cdot \dot{\vec{v}}) - \vec{v} \cdot \ddot{\vec{v}}$$

The time average of the total derivative vanishes. This defines the requirements to define time average

$$\overline{\frac{d}{dt}\vec{v}\cdot\dot{\vec{v}}} = \frac{1}{T}\int_0^T dt\,\frac{d}{dt}\,\vec{v}\cdot\dot{\vec{v}}$$

$$= \frac{1}{T}\left(\vec{v}_{(T)}\cdot\vec{a}(T) - \vec{v}(0)\cdot\vec{a}(0)\right)$$

$$= 0$$

Thus

$$\overline{\vec{F}_{rad}\cdot\vec{v}} = +\frac{q^2\mu_0}{6\pi c}\,\vec{v}\cdot\ddot{\vec{v}}$$

suggest

$$\vec{F}_{rad} = +\frac{q^2\mu_0}{6\pi c}\,\ddot{\vec{v}} = -\frac{q^2\mu_0}{6\pi c}\,\dot{\vec{a}}$$

Simplify notation

$$[\vec{F}] = m\dot{v} \qquad m =$$

$$\left[\frac{q^2\mu_0}{6\pi c}\right] = [m \times \text{time}]$$

$$\frac{q^2\mu_0}{6\pi c} \equiv m\tau$$

$$\tau = \frac{q^2\mu_0}{6\pi c m}$$

$$\boxed{P = m\tau\dot{v}^2}$$

Evaluate τ for electron

$$\tau = \frac{2}{3}\frac{q^2}{4\pi\epsilon_0}\frac{\mu_0\epsilon_0}{mc}$$

$fm = 10^{-15}\,m$

$$q^2/4\pi\epsilon_0 = \hbar c\,\alpha$$

, $\hbar c = 197.3$ MeV fm

$\alpha = \frac{1}{137}$

$$mc^2 = 0.511\ \text{MeV}$$

$\mu_0\epsilon_0 = \frac{1}{c^2}$

$$\tau = \frac{2}{3}\alpha\frac{\hbar c}{mc^3} = \frac{2}{3}\frac{1}{(137)}\frac{197\ \text{MeV}}{.511\ \text{MeV}}\frac{10^{-15}\,m}{c}$$

$$\approx 2 \times \frac{10^{-15}\,m}{c}$$

twice time it takes light to go 10^{-15} m

PDQ

$$\simeq 2 * 10^{-23}\ s$$

Such short time scales are in the quantum realm!!

Period of electron in Bohr atom $\sim 10^{-16}$ s $= \left(\frac{2\pi a_0}{137/c}\right)$

~~Now suppose~~ Electron subject to $\vec{F}_{ext}$ so eq. of motion must be written as

$$\boxed{m\left(\dot{v} - \tau\ddot{v}\right) = \vec{F}_{ext}} \quad \text{I}$$

Now this is a very strange equation

Suppose $\vec{F}_{ext} = 0$

There are many solutions: $\dot{\vec{v}} = 0$, $\ddot{\vec{v}} = 0$

But also

$$\boxed{\dot{v} = \dot{v}(0)\, e^{t/\tau}}$$

Check

$$\dot{v} - \ddot{v} = v_0 \left(1 - \frac{\tau}{\tau} \right) e^{+t/\tau} = 0$$

If particle starts from rest

$$V(T) = \int_0^T dt\, \dot{v}(0) e^{t/\tau} = \tau \dot{v}(0) \times e^{T/\tau}$$

$V(T)$ will quickly exceed speed of light

Nonetheless $\boxed{I}$ can be used if we are careful, <u>very careful</u>

Suppose <u>External</u> force acts for time T (short time) for which mean acceleration is $\dot{a}$

particle acquires kinetic energy $\sim m(\dot{v}T)^2$

radiates energy $\sim$ $m\tau \dot{v}^2 T$

Radiation reaction is small if

$$m\tau \dot{v}^2 T \ll m \dot{v}^2 T^2$$

$$\boxed{\tau \ll T}$$

If external force acts for a long time

radiation reaction is small

For classical situations

such as accelerating electrons

the radiation reaction may be neglected

QED gives the correct treatment