Follows from continuity of tangential components of $\mathbf{E}$ across boundary

$V_1 \sin \theta_2 \leq V_2 \leq V_1$ (Snell's Law)

Now apply boundary conditions. Continuity of

Subscripts 1, 2 refer to media

Normal D $\mathbf{E}_1 \cdot \mathbf{n}_1 = \mathbf{E}_2 \cdot \mathbf{n}_2 \rightarrow 0 = 0 \checkmark$

Normal B $\mathbf{B}_1 = \mathbf{B}_2 \quad \mathbf{B} \perp \mathbf{E}$

Sample incident +reflected

$E_0^I + E_0^R = \frac{V_1}{V_1} \sin \theta_2 \cdot E_0^I = \frac{1}{V_2} \sin \theta_2 \cdot E_0^R$ \( \square \)

Tangential $\mathbf{E}$ gives same eqn.

This gives

$\mu_1 \left[ \frac{1}{V_1} E_0^I (-\cos \theta_1) + \frac{1}{V_2} E_0^R \cos \theta_2 \right] ^T = \frac{1}{\mu_2} \frac{1}{V_2} \left( E_0^I (-\cos \theta_2) \right)$

$\Rightarrow \frac{E_0^I - E_0^R = \mu_1 V_1 \cos \theta_2 \cdot E_0^I}{\mu_2 V_2 \cos \theta_2}$

Two boxed equations are two eqns to two unknowns $E_0^I, E_0^R$ in terms of $E_0^I$

$\cot \beta = \frac{\cos \theta_2}{\cos \theta_1} \quad \beta = \frac{\mu_1 V_1}{\mu_2 V_2}$

Adding the 2 boxed equations gives

$E_0^R = \frac{2}{1 + \mu_1} E_0^I$
Given $E_{0r}$ use (1) to get $E_{0r}$

$$E_{0r} = \frac{1 - \alpha \beta}{1 + \alpha \beta} E_{0i}$$

$\alpha$, $\beta$ both positive

The two boxed equations are the Fresnel eq

Note $E_{0i}$, $E_{0r}$, $E_{t}$ are all complex numbers

since $\alpha \beta > 0$ $E_{0r}$ has the same phase as $E_{0i}$

$\alpha \beta \leq 1$ $E_{0r}$ has the same phase as $E_{0i}$

$\alpha \beta > 1$ $E_{0r}$ opposite phase

$$|E_{0r}| = \left| \frac{1 - \alpha \beta}{1 + \alpha \beta} \right| E_{0i}$$

(b) The Reflected intensity $= \frac{1}{2} I_{0r} |E_{0r}|^2$

Incident intensity $= \frac{1}{2} I_{0i} |E_{0i}|^2$

$$R = \frac{I_{0r}}{I_{0i}} = \left( \frac{1 - \alpha \beta}{1 + \alpha \beta} \right)^2 = R$$
(c) now \( d = \frac{\cos \theta_2}{\cos \theta_1} \) \( \beta = \frac{c}{v_2} = \frac{n - 1}{n + 1} \) where \( \varepsilon \) is small

\[ \theta_1 = \frac{\pi}{2} - \delta \]
\[ \delta < 1 \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

Snell's Law

\[ \sin \theta_1 = (1 + \varepsilon) \sin \theta_2 \]
\[ \sin \left( \frac{\pi}{2} - \delta \right) = (1 + \varepsilon) \sin \theta_2 \]

\[ \cos \delta = (1 + \varepsilon) \sin \theta_2 \]
\[ 1 - \frac{\delta^2}{\varepsilon^2} = (1 + \varepsilon) \left[ \cos \delta \right] = (1 + \varepsilon) \left( 1 - \frac{\delta^2}{\varepsilon^2} \right) \]

This justifies \( \delta < 1 \)

\[ \lambda = \frac{\cos \left( \frac{\pi}{2} - \theta_1 \right)}{\cos \left( \frac{\pi}{2} - \delta \right)} = \frac{\sin \lambda}{\delta} \approx \frac{\lambda}{\delta} \]

\[ \beta = 1 + \varepsilon \]

From 9.111\[ R_{11} = \left( \frac{d - \beta}{d + \beta} \right)^2 \]

\[ \left( \frac{X - \beta}{dX + \beta} \right)^2 = \left( \frac{Y - \beta}{dY + \beta} \right)^2 \]

From previous page \[ R_{11} = \left( \frac{1 - d \beta}{1 + d \beta} \right)^2 = \left( \frac{1 - \frac{\pi}{2} \beta}{1 + \frac{\pi}{2} \beta} \right)^2 \]
Multiply previous $R_1$ by $\delta / \beta$.

$$R_1 = \left( \frac{\delta / r - \beta}{\delta / \beta + \beta} \right)^2$$

$R_{11} \propto R_1$

These are the same if

$$\delta \gg \lambda$$

This is true if $\epsilon$ is small enough because from previous page

$$\gamma^2 = \delta^2 + 2 \epsilon$$

$\epsilon$ must be of order $\delta^3$ or higher power.