follows from continuity of tangential components of $\vec{E}$ across boundary

$v_1 \sin\theta_2 = v_2 \sin\theta_1$ (Snell's Law)

Now apply boundary conditions Continuity of

Subscripts 1,2 refer to media

Normal D $\quad \epsilon_1 \vec{E}_{1\perp} = \epsilon_2 \vec{E}_{2\perp} \quad \to 0 = 0$ ✓

Normal B $\quad B_{1\perp} = B_{2\perp} \quad \perp$ is in direction

$$\frac{1}{v_1} E_{0I} \sin\theta_1 + \frac{1}{v_1} E_{0R} \sin\theta_1 = \frac{1}{v_2} E_{0T} \sin\theta_2$$

sum of incident + reflected

$$E_{0I} + E_{0R} = \frac{v_1}{v_2} \frac{\sin\theta_2}{\sin\theta_1} E_{0T} = \boxed{E_{0T} = E_{0I} + E_{0R}} \quad (1)$$

Tangential E $\quad$ Eq. gives same Eqn.

Tangential H $\quad \frac{1}{\mu_1} \vec{B}_1^{\parallel} = \frac{1}{\mu_2} \vec{B}_2^{\parallel} \quad \parallel = \hat{x}$

This gives $\quad \frac{1}{\mu_1} \left[\frac{1}{v_1} E_{0I} (-\cos\theta_1) + \frac{1}{v_1} E_{0R} \cos\theta_1 \right]$

$$= \frac{1}{\mu_2} \frac{1}{v_2} E_{0T} (-\cos\theta_2)$$

$$\Rightarrow \boxed{E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} \frac{\cos\theta_2}{\cos\theta_1} E_{0T}} = \alpha\beta E_{0T}$$

Two boxed equations are two eqns w/ two unknowns E_{0R}, E_{0T} in terms of E_{0I}

Let $\alpha = \frac{\cos\theta_2}{\cos\theta_1} \quad \beta = \frac{\mu_1 v_1}{\mu_2 v_2}$

adding the 2 boxed eqns gives $\boxed{E_{0T} = \frac{2}{1+\alpha\beta} E_{0I}}$

Given E_{0T} use (1) to get E_{0R}

$$\boxed{E_{0R} = \frac{1-\alpha\beta}{1+\alpha\beta} E_{0I}}$$

α, β both positive

The two boxed equations are the Fresnel eq

Note E_{0I}, E_{0R}, E_{0T} are all complex numbers

since $\alpha, \beta > 0$ E_{0T} has the same phase as E_{0I}

if $\alpha\beta < 1$ E_{0R} has the same phase as E_{0I}

" (−) opposite phase "

if $\alpha\beta > 1$ E_{0R}

$$|E_{0R}| = \left|\frac{1-\alpha\beta}{1+\alpha\beta}\right| E_{0I}$$

(b) The reflected intensity $= \frac{1}{2}\epsilon_1 v_1 |E_{0R}|^2 \cos$

incident intensity $= \frac{1}{2}\epsilon_1 v_1 |E_{0I}|^2 \cos$

$$R = \frac{I_R}{I_I} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2 = R$$

(c) now $\alpha = \frac{\cos\theta_2}{\cos\theta_1}$ $\quad \beta = \frac{c}{v_2} = n = 1+\epsilon$

$\epsilon \ll 1$

where ϵ is small

$\theta_1 = \frac{\pi}{2} - \delta \qquad \delta \ll 1$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\quad$ Snell's Law

$\sin\theta_1 = (1+\epsilon)\sin\theta_2$

$\sin\left(\frac{\pi}{2} - \delta\right) = (1+\epsilon)\sin\theta_2$

$\cos\delta = (1+\epsilon)\sin\theta_2 \approx 1 - \frac{\delta^2}{2} = (1+\epsilon)\sin\theta_2$

$\theta_2 = \frac{\pi}{2} - \gamma$ with $\gamma \ll 1$? in question

$1 - \frac{\delta^2}{2} = (1+\epsilon)\left[\cos\gamma\right] = (1+\epsilon)\left(1 - \frac{\gamma^2}{2}\right)$

$-\frac{\delta^2}{2} = \epsilon - \frac{\gamma^2}{2}$ $\quad$ This justifies $\gamma \ll 1$

$$\alpha = \frac{\cos\left(\frac{\pi}{2} - \gamma\right)}{\cos\left(\frac{\pi}{2} - \delta\right)} = \frac{\sin\gamma}{\sin\delta} \approx \frac{\gamma}{\delta}$$

$\beta = 1 + \epsilon$

From 9.117 $\quad R_{11} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2$

$$= \left(\frac{\frac{\gamma}{\delta} - \beta}{\frac{\gamma}{\delta} + \beta}\right)^2$$

From previous page $\quad R_{11} = \left(\frac{1 - \alpha\beta}{1 + \alpha\beta}\right)^2 = \left(\frac{1 - \frac{\gamma}{\delta}\beta}{1 + \frac{\gamma}{\delta}\beta}\right)^2$

Multiply previous $R_{\parallel}$ by δ^2/γ^2

$$R_{\perp} = \left(\frac{\delta/\gamma - \beta}{\delta/\gamma + \beta} \right)^2$$

$R_{\parallel} \approx R_{\perp}$

These are the same, if

$\delta \approx \gamma$

This is true, if ϵ is small enough because from previous page

$$\gamma^2 = \delta^2 + 2\epsilon$$

ϵ must be of order δ^3 or higher power