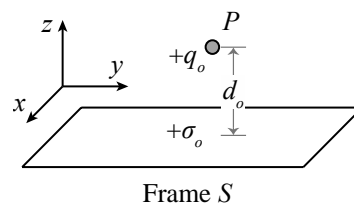V. [25 points total] Tutorial question.

In frame *S,* a positive test charge (labeled q_o) is held in place at point *P,* a distance d_o above an infinite sheet. The sheet is infinite in the *xy*-plane with charge density $+\sigma_o$ as shown at right.

Frame *S*

A. [4 pts] Determine $\vec{E}$ and $\vec{B}$ at point *P* due to the infinite sheet. Either specify both magnitude and direction, or express your answer as the field tensor.

$$\vec{E} = \frac{\sigma_o}{2\epsilon_o}\hat{z} \qquad \vec{B} = 0 \qquad \begin{pmatrix} 0 & 0 & 0 & \frac{\sigma_o}{2c\epsilon_o} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{-\sigma_o}{2c\epsilon_o} & 0 & 0 & 0 \end{pmatrix}$$

In frame *S',* the test charge is moves with constant velocity $\vec{u} = \frac{c\sqrt{3}}{2}\hat{y}$. At some instant, the test charge will pass point *P',* a distance d' above the infinite sheet.
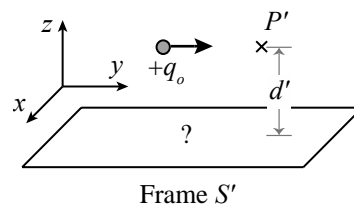
Frame *S'*

B. [4 pts] Determine d' in terms of d_o and/or defined constants. Explain your reasoning.

$d' = d_o$. Length measurements are not necessarily the same in different reference frames. However, length contraction only occurs in the direction of relative motion, so this particular length in not contracted as it is perpendicular to the direction of motion.

C. [5 pts] Circle the following components of $\vec{E}'$ and/or $\vec{B}'$ due to the sheet that are *non-zero* at point *P'.*

~~E_x'~~ ~~E_y'~~ E_z' B_x' ~~B_y'~~ ~~B_z'~~

Explain your reasoning.

- *The positively charged sheet transforms into a positively charged sheet with current in the $\hat{y}$-direction, which gives the fields above.*

- *Looking at the transforms, E_z transforms into both E_z' and B_x' when the boost is in $\hat{y}$. One can think of the boost as mixing the 0 and 2 indices, making $\tilde{F}_{03}$ into $\tilde{F}'_{03}$ and $\tilde{F}'_{23}$.*

D. [6 pts] Is there a new frame *S''* in which the B_z'' component of the field at point *P''* is non-zero? If so, describe a transformation (or series of transformations) to get to frame *S''* from frame *S.* Explain your reasoning.

If only Lorentz transforms are considered: No. Based on the Lorentz transforms, B_z'' will be in terms of B_z and a linear combination of E_x and E_y. None of these are present in the original system.

(However, a spatial rotation of the coordinates can turn B_x and/or B_y into B_z'', so a combination of a Lorentz transform to get a non-zero magnetic field and a spatial rotation to be able to label some component of the magnetic field as B_z'' would work.)

E. [6 pts] Is there a new frame *S'''* in which the charge density of the sheet is $+\frac{\sigma_o}{2}$? Explain your reasoning.

No. Based on the invariant squared norm $\left|\tilde{K}\right|^2 = -(\sigma_o c)^2$, the charge density must be at least σ_o since any current contributes positively to the squared norm. Also based on length contraction, the current-less charged plate can be considered "stationary," so any other frame would only see the charge spacing as contracted, increasing the density.