NAME:Solutions **Student ID:** **Score:**

Physics 323 **Spring 2017**

EXAM # 2
9 AM - 1020 AM, Thursday May 11

Write your name and ID number at the top of this page and on pages 2-5.

Clearly show all your reasoning.

You may use a calculator, but computers or other programmable devices are not allowed.

You are not allowed to use your phone during the exam.

This is a closed-book exam. Textbooks, class notes and other class material are not allowed.

Relevant formulae and equations are provided in a separate booklet.

Clearly note all constants and assumptions you use.

Show all your work and your final answers in the spaces provided. If you need to use the back of a page to complete your answer, clearly indicate this. Scratch work will not be graded.

Extra paper is available at the front of the classroom.

If you have a question during the exam, raise your hand.

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1. (25 pts total) *Retarded Potentials*
Suppose you take a thin plastic ring of radius a and place it on the $x-y$ plane, centered on the origin. You then glue charge on it, so that the line charge density is $\lambda_0|\sin(\gamma/2)|$ where γ is the angular coordinate about the ring. Then you spin the ring about its axis at an angular velocity ω. For parts (a-e), the field point is the ring center (origin).
(a) (4 pts) At time t, the ring element $\gamma = 0$ is located at angular position ϕ_0 on the $x-y$ plane. What is the retarded angular position for this ring element?

The retarded angular position is $\phi_r = \phi_0 - \omega a/c$.
(b) (6 pts) At time t, what is the scalar potential at the field point?

The distance to the field point is the same from all positions of the ring ($= a$). Thus the potential contributed by each ring element does not change and the scalar potential at the origin (field point) does not change over time, and is:
$V(0,t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}',t_r)}{a}d\tau' = \frac{1}{4\pi\epsilon_0}\frac{\lambda_0}{a}\int_0^{2\pi}|\sin(\gamma/2)|a d\gamma = \frac{\lambda_0}{4\pi\epsilon_0}\int_0^{\pi}2\sin(\theta)d\theta$, where we have used the subsitution $\theta = \gamma/2$. Evaluating the integral, we get:
$V(0,t) = \frac{\lambda_0}{\pi\epsilon_0}$.
(c) (7 pts) At time t, what is the vector potential at the field point? Evaluate in Cartesian coordinates. You may leave your answer in integral form, but simplify as much as possible.

We have $\mathbf{A}(0,t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}',t_r)}{a}d\tau'$, where again the distance to the field point is not changing. Working in Cartesian coordinates, we can determine the current in the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ directions separately. For the $\hat{\mathbf{x}}$ direction, the linear current is given by the rate of change of charge in the x-direction. Thus $Q_x = \lambda_0\sin(\gamma/2)a\cos(\gamma+\omega t)$ and $I_x = \dot{Q}_x = \lambda_0 a\omega\sin(\gamma/2)(-\sin(\gamma+\omega t))$. Similarly, $Q_y = \lambda_0\sin(\gamma/2)a\sin(\gamma+\omega t)$ and $I_y = \dot{Q}_y = \lambda_0 a\omega\sin(\gamma/2)(\cos(\gamma+\omega t))$. Noting that $\int \mathbf{J}(\mathbf{r}',t_r) = \int_0^{2\pi}(I_x(\gamma,t_r)\hat{\mathbf{x}} + I_y(\gamma,t_r)\hat{\mathbf{y}})a d\gamma$ and $t_r = t - a/c$, we have:
$\mathbf{A}(0,t) = \frac{\mu_0\lambda_0 a\omega}{4\pi}\int_0^{2\pi}(-\sin(\gamma/2)(\sin(\gamma+\omega t_r))\hat{\mathbf{x}} + \sin(\gamma/2)(\cos(\gamma+\omega t_r))\hat{\mathbf{y}})d\gamma$
(d) (4 pts) Explain if you can use the results of parts (b) and (c) to find $\mathbf{E}$ and $\mathbf{B}$ at the origin.

We have $\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$. While the second term can be evaluated as the time derivative of the result of (b), the first requires knowledge of the spatial variation of V near the origin, something not detemined in (b) or (c).
We have $\mathbf{B} = \nabla\times\mathbf{A}$. This too cannot be evaluated based on the results of (b) and (c), since the spatial variation of $\mathbf{A}$ is required, not just the single value at the origin. Thus neither the electric or magnetic field at the origin can be found using the results of (b) and (c). More work needs to be done to evaluate the potentials around the origin to obtain their gradients and curls.
(e) (4 pts) Find a non-zero value of ω for which the retarded vector potential is identical to that calculated in the instantaneous limit (ie, $a/c \to 0$).

With respect to the origin, this condition is satisfied when the difference between retarded and present time is some multiple of the period. Thus $\frac{2\pi}{\omega} = \frac{a}{c}$, implying $\omega = \frac{2\pi c}{a}$ or a multiple.

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2. (25 pts total) *Radiation: KOMO TV*
KOMO TV transmits with 1 MWatt radiated power at a frequency near 600 MHz from Queen Anne Hill in Seattle. A simplified version of the KOMO transmitting antenna consists of a vertical wire, with a small gap between the wire and the earth. The earth acts as a ground-plane for the radiating system. The transmitter cable applies the signal across the earth and the wire.
(a) (5 pts) As seen from large distances from the antenna, in which directions are nulls of the radiation pattern and in which directions are maxima of the radiation pattern?

In the far field, a conventional dipole antenna radiates in a dipole pattern with intensity proportional to $\sin^2\theta$ where θ is the polar angle about the antenna axis. Since this is a half-antenna, we can treat this by the image method to find that the radiation above ground is identical to the conventional $\sin^2\theta$ variation, while there is no field below the earth (ground-plane). Thus, at large distances:
Nulls are directly above the antenna axis and everywhere below the ground.
Maxima are immediately above ground everywhere around the antenna.

(b) (5 pts) For a receiving antenna near ground level at a large distance from the antenna, describe the electric and magnetic field polarizations.

The electric field polarization is vertical. The magnetic field polarization is horizontal and in the direction which satisfies $\mathbf{E} \times \mathbf{B} \propto \hat{\mathbf{r}}$ where $\hat{\mathbf{r}}$ is the unit vector pointing from the transmitting antenna to the receiving antenna.

(c) (5 pts) Explain why KOMO doesn't use a conventional electric dipole antenna, but instead chose half an electric dipole antenna with the earth as a ground-plane?

Understanding the problem using the image method (as in (a)), we see that there is no radiation below the ground plane. The half-antenna radiates only to all customers above ground. This way KOMO can keep its power cost down by a factor of two.

(d) (5 pts) Suppose the signal amplitude in the transmitter is kept the same, but the frequency is increased to 900 MHz. How much power will be radiated?

In the far field, the radiated power goes as ω^4. Thus, radiated power will be 1 MWatt $\times(\frac{900}{600})^4$ or about 5 MWatts.

(e) (5 pts) Explain in 20 words or less: How can the EM waves transmitted by different TV stations be received on a customer's antenna without any problematic effects from wave interference?

Different stations transmit at different frequencies. This allows isolation by frequency filtering or tuning at the receiver end.

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3. (25 pts total) *Potential Formulation*
Suppose $V = 0$ and $\mathbf{A} = A_0\sin(kx - \omega t)\hat{\mathbf{y}}$, where A_0, ω, and k are constants.
(a) (10 pts) Find $\mathbf{E}$ and $\mathbf{B}$.

Using potential formulation of fields:
$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = 0 - A_0(-\omega)\cos(kx - \omega t)\hat{\mathbf{y}} = A_0\omega\cos(kx - \omega t)\hat{\mathbf{y}}$
In $\mathbf{B} = \nabla \times \mathbf{A}$, only the $\frac{\partial A_y}{\partial x}$ term will contribute (the others are zero). Thus:
$\mathbf{B} = \nabla \times \mathbf{A} = \frac{\partial A_y}{\partial x}\hat{\mathbf{z}} = A_0 k\cos(kx - \omega t)\hat{\mathbf{z}}$;

So we have:
$\mathbf{E} = A_0\omega\cos(kx - \omega t)\hat{\mathbf{y}}$ and $\mathbf{B} = A_0 k\cos(kx - \omega t)\hat{\mathbf{z}}$.

(b) (10 pts) Show that $\mathbf{E}$ satisfies the free-space (vacuum) wave equation.

$\mathbf{E}$ has only a y-component. Thus $\nabla^2\mathbf{E} = (\nabla^2 E)\hat{\mathbf{y}} = -A_0\omega k^2\cos(kx - \omega t)\hat{\mathbf{y}}$.
We also have $\frac{\partial^2 \mathbf{E}}{\partial t^2} = -A_0\omega^3\cos(kx - \omega t)\hat{\mathbf{y}}$.
Thus:
$\nabla^2\mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$ and $\mathbf{E}$ satisfies the free-space wave equation. Here we must stipulate that ω and k need to be related as $\omega = ck$, the usual free-space dispersion relation.

(c) (5 pts) In which direction does this wave propagate?

The wave propagates in the direction given by $\mathbf{E} \times \mathbf{B}$. From (a), $\mathbf{E}$ points in the $\hat{\mathbf{y}}$ direction and $\mathbf{B}$ points in the $\hat{\mathbf{z}}$ direction. Thus, $\mathbf{E} \times \mathbf{B}$ points in the $\hat{\mathbf{x}}$ direction.
Thus, we conclude that the wave propagates in the $\hat{\mathbf{x}}$ direction.

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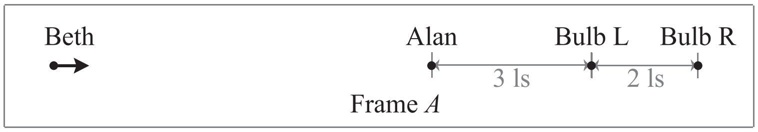
last *first*

IV. [25 points total] Tutorial question.



Alan stands at the origin of frame *A*. Two stationary light-bulbs L and R are located 3 and 5 light-seconds along the *x*-axis, respectively. Both light-bulbs flash, and Alan sees the light from both light-bulbs at the same time.

A. [3 pts] Did light-bulb L flash *before, at the same time as,* or *after* light-bulb R? Explain.

After. Alan seeing the light is not the same as the bulbs flashing. Light from bulb L takes less time to travel to Alan (less distance covered going the speed of light) than the light from bulb R, so in order to end at Alan at the same time, bulb L had to have flashed after bulb R.

B. [4 pts] Is the retarded time of the flash from light-bulb L *greater than, less than,* or *equal to* the retarded time of the flash from light-bulb R? Explain your reasoning.

Greater than. One can interpret retarded time from the observer to the source as the past time of the source as the observer is viewing the source. This question is essentially asking part A again, so the retarded time of the flash from bulb L is greater, as bulb L flashed later.

Mathematically, using $t_{ret} = t - \frac{d}{c}$*, Alan sees both flashes at the same time (t is equal), but bulb L is closer than bulb R, so the retarded time for the flash from bulb L is greater.*

C. Beth is in frame *B,* which is moving at relativistic speeds in the +*x*-direction in frame *A*. At the time that Alan sees the light from the light-bulbs, Beth is located somewhere on his –*x*-axis. You may answer the following parts in any order and reference other answers for explanation if necessary.

i. [6 pts] Is the measured distance between the two lightbulbs in frame *B greater than, less than,* or *equal to* 2 light-seconds? Explain your reasoning.

Less than. The measured distance can be considered a length, because this measurement in a frame must be instantaneous. Because frame A is the "rest frame" of the bulb separation, all frames moving with an x-component of velocity will measure this length to be contracted.

ii. [6 pts] Does Beth see light from light-bulb L *before, at the same time as,* or *after* light-bulb R? If there is not enough information, state so explicitly. Explain your reasoning.

At the same time. By thinking of the light from the flash as projectiles of light (photons), we can say that both photons must travel together once they pass bulb L, in order to travel at the same speed and reach Alan at the same time. They will then travel together and hit Beth at the same time. Since the trajectories are not separated by space or time, all reference frames will observe that the photons travel together and hit Beth at the same time.

iii. [6 pts] Is the time difference between the two flashes measured in frame *B,* Δt_B*, greater than, less than, or equal to* Δt_A*?* If either or both differences are zero, state so explicitly. Explain.

Greater than. The two flashes do not occur in the same location in frame B, so one cannot simply use time dilation (the factor is incorrect, as explained below). Based on the previous explanation, Δt *can be thought of as the time it takes for light from bulb R to reach bulb L. This distance has been contracted by a factor of* $1/\gamma$*, but because bulb L is moving toward Beth after the flash of bulb R, the relative rate at which light covers this distance is* $c - v$*. By relating* $\Delta t = \frac{\Delta x}{c - v_L}$*, the relative speed has been decreased more than the length has been contracted, so the time difference increased.*