IV. [20 points total] Tutorial question.

Consider an electromagnetic wave with the complex electric field vector given by:

\[ \vec{E} = \frac{E_o}{r} e^{-kr} e^{i[kr+(1+i)\omega t]} \hat{\phi} \]

where \( E_o, k, \) and \( \omega \) are real, positive numbers.

A. [6 pts] Describe and/or sketch the wavefronts of this wave at a single instant in time. Explain your reasoning.

The wavefronts look like concentric spheres around the origin. Wavefronts correspond to constant values of argument in the complex exponential, or the “\( kr \)” in \( e^{i[kr+(1+i)\omega t]} \). At a single instant in time, having the same \( r \) gives the same argument, which describe spheres.

B. Consider the time dependence of this wave.

i. [5 pts] In what direction(s) does this wave propagate? Explain your reasoning.

The wave propagates inward, in \(-\hat{r}\). The relative sign between the space and time terms in \( e^{i[\omega t]} \) tells us the direction of propagation. As time progresses, \( r \) must decrease in order to keep the argument the same.

ii. [4 pts] Describe in words the behavior of the electric field at a single point in space. Explain your reasoning.

The magnitude of the electric field cycles between maximum, zero, and minimum while decreasing over time. \( e^{i[(1+i)\omega t]} \) simplifies to \( e^{i\omega t} e^{-\omega t} \), where the complex exponential shows cyclic behavior, but the real exponential shows decay.

C. [5 pts] Under what condition(s) can this wave be approximated as a plane wave? If this already describes a plane wave or cannot be approximated as a plane wave, state so explicitly. Explain your reasoning, ensuring that you accurately describe what makes a wave a plane wave.

The condition is that \( \Delta r \ll r \), or “very far from the origin.” A “plane” wave must have planar geometry, such that the wavefronts look like infinite planes in two Cartesian directions. When the spheres get big enough relative to the positions being compared, the curve of the sphere becomes negligible so the surface of the sphere looks planar.