PHYSICS 323:
ELECTROMAGNETISM

21 April 2020 Midterm 1

Name: ______________________________ Stu. ID. ____________________

Section number ______________________________

There are four problems in this exam, each with several parts. Each problem is worth 20 points.

This exam is open book, open notes, open internet, open calculator, and open computer, but the work must be your very own. Therefore you must sign your name below.

I certify that the submitted answers are my own work, done without collaboration

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Sign your name on the line above.

Read the problems carefully.

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1. *Plasma Physics.*

A plasma is an ionized gas consisting at least partly of free electrons (of mass m) and positively charged ions; it therefore can be a conducting material. There are no bound dipole moments, and the medium is non-magnetic ($\epsilon = \epsilon_0, \mu = \mu_0$). The sun and stars are largely plasmas. Consider the classical model discussed in class and in Sect. 9.4.3, with $\gamma = 1/\tau$, and the electrons are not bound ($\omega_0 = 0$). You may ignore the effects of any positively charged ions.

(a) (10) The density of the electrons in the plasma is N and the current density $\mathbf{J} = -eN\mathbf{v} = \sigma\mathbf{E}$. Take the electric field to have the form $\mathbf{E}(t) = \mathbf{E}_0 e^{-i\omega t}$ Show that the conductivity is given by $\sigma(\omega) = \frac{i\epsilon_0\Omega^2}{\omega+i\gamma}$ with $\Omega^2 \equiv \frac{Ne^2}{m\epsilon_0}$.

(b) (5) Now consider a plane wave with the form $\mathbf{E}(t) = \mathbf{E}_0 e^{i(kz-\omega t)}$ traveling through a non-magnetic medium with the stated $\sigma(\omega)$. Compute the value of k (appropriate for propagation in the positive z direction) in terms of the given quantities. You may express your answer in terms of a well-defined square root.

(c) (5) For a dilute plasma the damping factor γ is very small, so take $\gamma = 0.01\Omega$, and for a typical plasma $\Omega = 1.5 \times 10^{10}$ rad/s. For what frequencies, ω, is $\mathrm{Re}(\sigma) \gg \mathrm{Im}\ (\sigma)$.

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2. *Total reflection* Consider the case that light propagates from a dense optical medium, $n > 1$, into air with $n = 1$. See Fig. 9.14, with the left-hand side of $n > 1$ and the right-hand side with air, $n = 1$. Both sides have $\mu = \mu_0$.

(a) (5) For what angles θ_I is the angle of transmission (angle of refraction) θ_T a complex number?

(b) (10) Consider the wave that moves in the air for situations in which θ_T is a complex number. Show that the wave propagates along the x-direction, but falls off exponentially in the z direction.

(c) (5) Compute the ratio of the intensity of the reflected to that of the incident wave for the case of incident waves polarized parallel to the plane of incidence.

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3. *Cylindrical wave guide* Consider a hollow cylindrical wave guide of radius a. The cylinder is made of a perfect conductor. The aim is learn about the TM modes that propagate down the cylinder in the z direction. Use cylindrical coordinates and write $E_z(\mathbf{r}, t) = \psi(s, \phi)e^{i(kz-\omega t)}$.

(a) (5) What is $\psi(a, \phi)$?

(b) (8) Explain why a solution $\psi(s, \phi)$ can be written as $R(s)e^{im\phi}$ with m an integer (1), and obtain a differential equation for $R(s)$(7).

(c) (7) Find the lowest cutoff frequency.

The regular solutions of the equation $\frac{d^2R}{dx^2} + \frac{1}{x}\frac{dR}{dx} + (1 - \frac{m^2}{x^2})R = 0$ are denoted Bessel functions $J_m(x)$.

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last *first*

IV. [20 points total] Tutorial question.

Consider an electromagnetic wave with the complex electric field vector given by:

$$\tilde{E} = \frac{E_o}{r} e^{-kr} e^{i[kr+(1+i)\omega t]} \hat{\phi}$$

where E_o, k, and ω are real, positive numbers.

A. [6 pts] Describe and/or sketch the wavefronts of this wave at a single instant in time. Explain your reasoning.

B. Consider the time dependence of this wave.

i. [5 pts] In what direction(s) does this wave propagate? Explain your reasoning.

ii. [4 pts] Describe *in words* the behavior of the electric field at a single point in space. Explain your reasoning.

C. [5 pts] Under what condition(s) can this wave be approximated as a plane wave? If this already describes a plane wave or cannot be approximated as a plane wave, state so explicitly. Explain your reasoning, ensuring that you accurately describe what makes a wave a plane wave.