# PHYSICS 322 ELECTROMAGNETISM

23 Feb. 2020 Problem Set 6 These **four** problems are due **in your tutorial box by 11am Tuesday, March 3 Please put your name and section number on the first page of your solutions. The graded homework will be returned in your sections.**

1. *Point Charge and Solenoid*

(a) A point charge $q$ is located at a position $(x, y, z) = (d, 0, 0)$. An infinite solenoid with $n$ turns/length, radius $R$, $(d \gg R)$, and carrying steady current $I$ is positioned along the $z$-axis. Find the linear momentum and angular momentum (about the origin) in the fields.

(b) Now consider the same situation as in part (a), but take the current to have a time-dependence $I = 0$, $t < 0$, $I = I_0 \frac{t}{T}$, $0 \leq t \leq T$, $I = I_0$, $t > T$, where $T$ is a given time. For $0 \leq t \leq T$, determine the linear momentum and angular momentum (about the origin) in the fields.

2. *Current continuity* An infinite cylindrically-shaped wire runs along the $z$ axis; it carries a current $I(z)$ that is a function of $z$ (but not of $t$), and a charge density $\lambda(t)$ that is a function of $t$ (but not of $z$).

(a) By examining the charge flowing into a segment $dz$ in a time $dt$, show that $d\lambda/dt = -dI/dz$. Suppose $\lambda(0) = 0$ and $I(0) = 0$, show that $\lambda(t) = kt$, $I(z) = -kz$, where (k) is a constant.

(b) Assume for a moment that the process is quasistatic, so the fields outside the wire are as given by Eqs. 2.9 and 5.38 of Griffiths. Show that these are in fact the exact fields (for the situation of (a)) , by confirming that all four of Maxwell's equations are satisfied. This problem is meant to justify not using the full Maxwell equations until chapter 7.

3, Maxwell Stress Tensor Two point charges, each of charge $q$ are a distance $2d$ apart in free space.

(a) Find the Maxwell stress tensor on a plane surface midway between the charges.

(b) Find the force on either charge by integrating $dA_n T_{ni}$ over a plane surface, closing the surface with a very large hemisphere of radius $R$. Show that the integral over the hemisphere vanishes in the limit $R \to \infty$.

(c) Find the force on the charge at $-\mathbf{d}$ if the charge at $+\mathbf{d}$ is $-q$.

4. *Power emitted by a wire* A long straight copper wire, resistivity $\rho$, of radius $a$ and length $L$ carries a constant current $I$.

(a) Find the electric and magnetic fields at the surface of the wire.

(b) Integrate the Poynting power flux through the surface of a piece of the wire of length $L$ to show that the power into the surface is $I^2 R$.

(c) Find the electromagnetic energy and momentum in this piece of the wire.