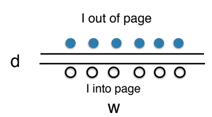
PHYSICS 322 ELECTROMAGNETISM

10 Feb. 2020 Problem Set 5 These **four** problems are due **in your tutorial box by 11am Tuesday, 12 Feb. Please put your name and section number on the first page of your solutions. The graded homework will be returned in your sections.**

1. *Magnetic energy* Equal but opposite currents I flow on the top and bottom of two long parallel plates, as shown in the figure. The plates have width w separation d, where $d \ll w$.



(a) Neglecting edge effects, find the magnetic field between the two plates.

(b) Calculate the magnetic field energy per unit length.

(c) Compute the self inductance per unit length.

2. *Motion of loop inductor in magnetic field* A square loop of side length d has mass m, inductance L and zero resistance, $R = 0$. It is placed in the xy plane, centered at the origin and with its sides parallel to the x and y axes. The magnetic field in this region of space can be described by $\mathbf{B} = B_0\hat{k}$ for $y \leq 0$, $\mathbf{B} = 0$ for $y > 0$. The loop is given an initial velocity $v = v_0\hat{\jmath}$, $v_0 > 0$. Hint: see Example 7.12.

(a) Initially, which direction does the current in the loop start to flow?

(b) Which direction in the initial force on the loop?

(c) By relating the force to the current which in turn is related to the change in flux, derive a differential equation for v.

(d) Solve this differential equation to determine the motion of the loop at later times.

(e) Determine the peak current generated in the loop.

3. *Conduction vs Displacement currents*

At frequency $\omega = 2\pi \times 400$ MHz, sea water has permittivity $\epsilon = 81\epsilon_0$, permeability $\mu = \mu_0$ and resistivity $\rho = 0.23\,\Omega$-m (SI units). A sample of sea water is threaded by radiofrequency (RF) radiation (see Page 396) described by the alternating electric field $E_0 \cos(\omega t)$. Here conduction current describes the usual current (as opposed to displacement current) and refers to the $\mathbf{J}$ that enters the Maxwell equation for $\boldsymbol{\nabla} \times \mathbf{B}$

(a) Determine the ratio of the magnitude of the conduction current to the magnitude of the displacement current.

(b) Is the conduction current in-phase or out-of-phase with the electric field? If out-of-phase, by how much does the conduction current lead or lag (specify which)?

(c) Is the displacement current in-phase or out-of-phase with the electric field? If out-of-phase, by how much does the current lead or lag (specify which)?

4. *Charging a capacitor, boundary conditions and displacement current*

(a) Show that the discontinuity of $\mathbf{B}$ across a circular capacitor plate of radius a as the capacitor is being charged with current I is equal to $\mu_0 \mathbf{K} \times \hat{\mathbf{n}}$, where

$$\mathbf{K} = \frac{\hat{\mathbf{s}} I}{2\pi} \left(\frac{1}{s} - \frac{s}{a^2} \right).$$

(b) By the general boundary condition on the tangential field, $\mathbf{K}$ is the surface current density that flows radially outward as the plate becomes charged. Show that the above form of the surface current implies that the charge remains uniformly distributed over the area of the plate. (Hint: Show that the rate of change of charge in an annulus (thin ring) at an arbitrary radius is proportional to the area of the annulus. Therefore, the charge divided by the area is constant.)