PHYSICS 322 ELECTROMAGNETISM

20 Jan. 2020   Problem Set 3   These problems are due in **tutorial boxes Tuesday, 4 Feb.**

Please put your name and section number on the first page of your solutions. The graded homework will be returned in your sections.

1. *Rotating a Polarized Cylinder* An infinitely long cylinder of radius R has a permanent polarization $\mathbf{P}$ (dipole moment per unit volume) in the radial direction from the axis of the cylinder. . The magnitude of the polarization is $P_0 s$ with P_0 a constant and s the distance from the axis.

(a) Find the bound surface and volume charges.

(b) Find the electric field inside and outside the cylinder.

(c) Now rotate the cylinder about its axis at constant angular velocity ω Find the magnetic field inside and outside the cylinder. The frequency $\omega R \ll c$, so that the condition of steady currents is maintained.

2. *Magnetic dipole-dipole interaction* Consider two current loops, each of radius R carrying a current I. The centers are a distance d apart, with $d \gg R$.

(a) Take the first loop (1) to lie in the xy-plane with center at the origin. The second loop (2) also lies in the xy plane, and is centered at $y = d$. Both currents are directed in the count-clockwise sense around the z axis. Determine the force of one of the current loops on the other. Are your results consistent with Newton's third law?

(b) Now consider the situation in which one loop (1) is centered at the origin and lies in the xz plane. The other loop (2) lies in the xy plane, with its center at $y = d$ The direction of the currents are clockwise about the y-axis for the (1) and counter-clockwise about the z-axis for the loop (2). Determine the force of one of the current loops on the other. Are your results consistent with Newton's third law?

3 *Cylinder with non-uniform axial magnetization* An infinitely long cylinder of radius R , carries a "frozen-in" magnetization, parallel to the axis $\mathbf{M} = (k_0 + k_1 s + k_2 s^2)\hat{\mathbf{z}}$, where k_0, k_1, k_2 are constants and s is the distance from the axis. There is no free current anywhere. Find the magnetic field inside and outside the cylinder by two different methods:

(a) Locate all the bound currents and calculate the field they produce.

(b) Use Ampere's law for magnetized materials to find $\mathbf{H}$, and then get $\mathbf{B}$ from the relationship between $\mathbf{B}, \mathbf{H}, \mathbf{M}$.

4. *Current carrying wire in water* A. very long straight wire of circular cross section with radius a carries current I and is immersed in a large volume of water. The wire is non-magnetic.

(a) Compute the magnetization in $\mathbf{M}(\mathbf{r})$ in water.

(b) Determine the bound current density $\mathbf{J}_b(\mathbf{r})$ and compute the *total* current (free plus bound). Possibly useful hint: use $\mathbf{J}_b(\mathbf{r}) = \boldsymbol{\nabla} \times \mathbf{M}$ and the derivative of the theta function.