PHYSICS 322 ELECTROMAGNETISM

12 Jan. 2020 Problem Set 2 These problems are due **in tutorial box on Tuesday, 21 Jan.**

Please put your name and section number on the first page of your solutions. The graded homework will be returned in your sections.

1 *Using the vector potential* Consider a very long straight wire carrying a current I that points in the positive $z-$ direction. The wire has length L which may be taken as $L \gg r$, where r is the distance from any point on the wire.

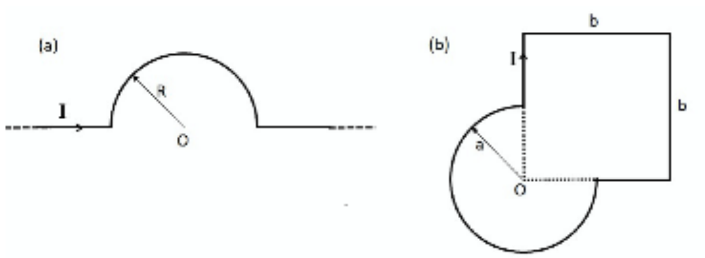
(a) Determine the vector potential $\mathbf{A}$.

(b) Use the vector potential you compute to obtain the magnetic field B Hint: if you find that $\mathbf{B}$ vanishes go back to part (a).

(c) Check that the divergence $\mathbf{A}$ vanishes.

(d) The general vanishing of $\mathbf{\nabla} \cdot \mathbf{A}$ is a consquence of magnetostatics (not merely a choice of gauge) when the current distribution is localized. Prove this statement.

2. (a) A planar current path consists of a pair of infinitely long wires connected to a semi-circle of radius R, as shown in figure (a). What is the magnetic field at the center point O of the semicircle when current I flows in the path from left to right?
(b) A planar current path consists of a circular arc containing three quadrants of a circle (radius a) connected to a part of a square loop (side length b), as shown in figure (b). The "missing corner" of the square is located at the center O of the circular arc. What is the magnetic field at O when current I flows in the path in a clockwise manner?



3. *Boundary conditions* A steady current I flows down a long cylindrical wire of radius R.

(a) Determine the magnetic field, inside and outside the wire, if all the current is uniformly distributed over the **outside surface** of the wire only.

(b) Determine the magnetic field, both inside and outside the wire, if the current is distributed in such a way that the current density $\mathbf{J}$ is proportional to s^2 where s is the distance from the axis: $\mathbf{J} = J_0 \frac{s^2}{R^2} \hat{\mathbf{k}}$.

(c) Show that your answers to (a) and (b) are consistent with the magnetostatic boundary conditions (Griffiths Eqn (5.76)) at the outside surface $s = a$.

4. Magnetic forces on currents Two circular current loops of radius R, each carrying a current, I, (in the same direction) are a distance $L = 4R$ apart on the same axis. Find the force on the two coils by applying the magnetic dipole field of the bottom coil to the upper coil. Hint: consider the horizontal and vertical components of the magnetic field separately.