Welcome to PHYS 322

web page http://faculty.washington.edu/miller/32220

Magnetism

Magnets are important in every day lives. Needed for

electric motors - magnetic torque rotates a moving arm

TV picture tubes - magnets guide electron beam

computer disk drives

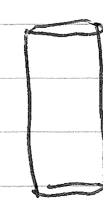
Magnetic phenomema discovered 2500 Years ago

Demo - magnets pickup metal

Fundamental force Law

point charge Q

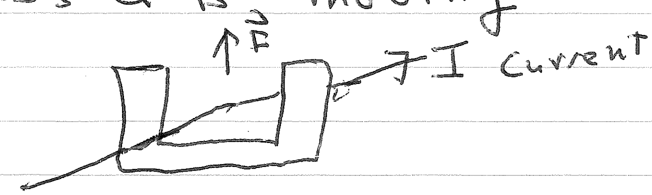
Q •



Magnet

No force unless Q is moving

Jumping wire



Strength of magnetism denoted by $\vec{B}$, the magnetic field

Wires carrying current generate $\vec{B}$ - Demo Magnetic fields around conductors

Basic Force Law

$$\vec{F}_{mag} = Q(\vec{v} \times \vec{B})$$

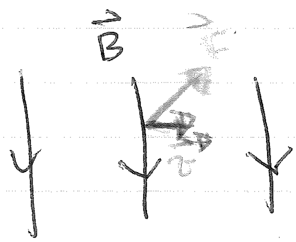
$\vec{F}_{mag}$ in direction $\perp \vec{v}$ + $\perp \vec{B}$

If there is also an electric field

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{Lorentz force law}$$

UNITS $[vB] = [E]$

Examples of motion



1. Q at rest what is force

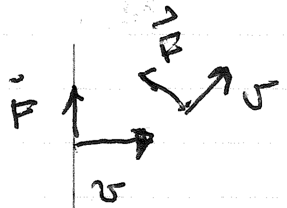
2. Suppose $\vec{v} \perp \vec{B}$ what is $\vec{F}$

Does speed change? $\vec{F} \cdot d\vec{s} = dW = d\text{Kin. Energy}$

Compute motion $F = QvB$

$\vec{F} \perp \vec{v}$ this is condition for circular motion

$$\vec{F} = m\vec{a} \qquad a = \frac{v^2}{R}$$



$$QvB = m\frac{v^2}{R}$$

$mv = QBR$ cyclotron motion

3. Suppose $\vec{v}$ is not $\perp$ to $\vec{B}$

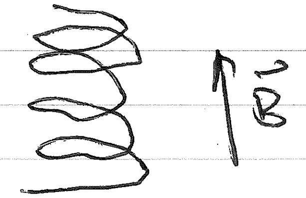
$$\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$$

The only Mag. force is due to $\vec{v}_{\perp}$

Q particle moves in circle

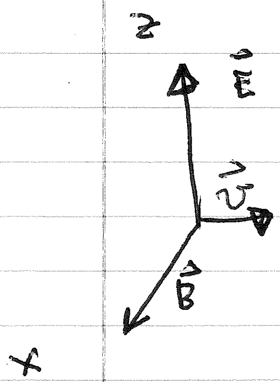
but the circle moves with $\vec{v}_{\parallel}$ along

the direction of the $\vec{B}$. This

motion  is called a helix

4. Next do $\vec{E}$ & $\vec{B}$ with $\vec{E} \perp \vec{B}$

$\vec{E} = E_0 \hat{z}$, $\vec{B} = B_0 \hat{x}$ $\quad Q > 0$



particle moves in y direction with speed v

what happens?

what is direction of Force? z-

$$F_z = Q(E - vB)$$

particle accelerates in z direction unless $v = E/B$, then particle moves with constant speed

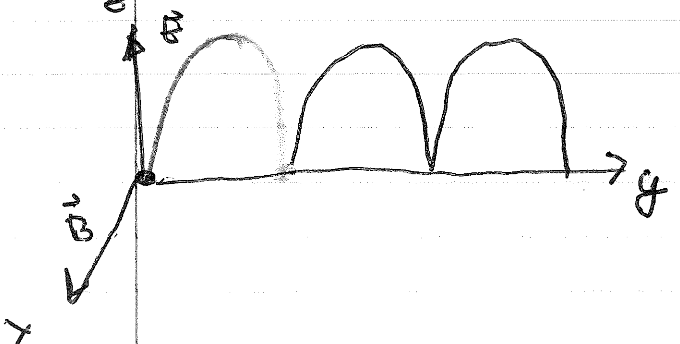
This set up is called a velocity selector particle released from rest

More interesting - $\vec{E} = E_0 \hat{z}$

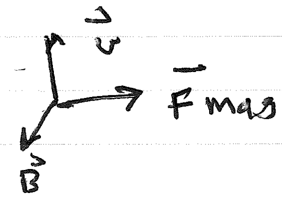
$= -\vec{\nabla} V$

$V = E_0 z$

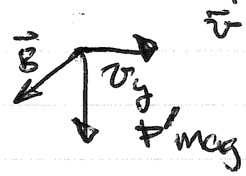
depends on z only



$Q > 0$ released from <u>rest</u>, initial force is upward, then have



particle accelerates to right. Then $\vec{v}$ has a horizontal component



New mag force downward

The Q returns to xy plane, $z = 0$

what is speed then?

Compute work done by forces

$$dW = \vec{F} \cdot d\vec{l} = \vec{F} \cdot \vec{v}\, dt$$

$\vec{F}_{mag} \perp \vec{v}$

$$= Q\vec{E} \cdot \vec{v}\, dt$$

<u>only work is electrostatic</u>

But particle has returned to same height. $\Delta V = 0$ No work has been done. No change in kinetic energy when $z=0$ $\quad v=0$.

This is almost the same as the initial condition

only diff. is translation along y axis

So pattern repeats

Now words give us a picture but we

want to describe position quantitatively

use $\vec{F}=m\vec{a}$

$$\vec{v}=(0,\dot{y},\dot{z}) \quad \text{with } \dot{y}=\frac{dy}{dt}$$

$$\dot{z}=\frac{dz}{dt}$$

$$\vec{v}\times\vec{B}=\begin{vmatrix} \hat{x} & \hat{j} & \hat{k} \\ 0 & \dot{y} & \dot{z} \\ B & 0 & 0 \end{vmatrix}$$

$$= \hat{j} \quad B\dot{z} - B\dot{y}\hat{k}$$

$$\vec{F} = Q\vec{E} + Q\vec{v}\times\vec{B} \qquad |\vec{E}|=E$$

$$F_z = QE - QB\dot{y} = m\ddot{z}$$

$$F_y = QB\dot{z} = m\ddot{y}$$

So
$$\boxed{\begin{aligned} \ddot{y} &= \frac{QB}{m}\dot{z} = \omega\dot{z} \\ \ddot{z} &= \frac{QE}{m} - \omega\dot{y} \end{aligned}}$$

$$\left[\frac{QB}{m}\right] = \frac{1}{\text{time}} = [\omega]$$

The box is a coupled differential equation

differentiate the first to get

$$\dddot{y} = \omega \ddot{z} = \omega\left(\frac{QE}{m} - \omega \dot{y}\right)$$

$$\dddot{y} + \omega^2 \dot{y} = \frac{\omega Q E}{m}$$

$$\left(\frac{d^3}{dt^2} + \omega^2\right)\dot{y} = \frac{\omega Q E}{m}$$

$$\dot{y} = \dot{y}_H + \dot{y}_1$$

with $\left(\frac{d^3}{dt^2} + \omega^2\right)\dot{y}_H = 0$, $y_H \Rightarrow C_1 \sin \omega t + C_2 \cos \omega t$

and

$$\left(\frac{d^3}{dt^2} + \omega^2\right)\dot{y}_1 = \frac{\omega Q E}{m}$$

$$\dot{y}_1 = \frac{QE}{m\omega}$$

$$y_1 = \frac{QEt}{m\omega} + C_3$$

Thus $y = C_1 \sin \omega t + C_2 \cos \omega t + \frac{QEt}{m\omega} + C_3$

3 constants - as expected from 3rd order Diff Eq

$y(t=0) = 0 \Rightarrow C_2 + C_3 = 0$

$\dot{y}(t=0) = 0 = (+C_1 \omega \cos \omega t - C_2 \omega \sin \omega t) + \frac{QE}{m\omega}$

take $t = 0$

$$0 = C_1 \omega + \frac{QE}{m\omega}$$

$$\ddot{y}(t=0) = \omega \dot{z}(t=0) = 0$$

$$= -\omega^2 C_1 \sin\omega t - \omega_2 C_2 \cos\omega t = 0$$

So $C_2 = 0$
and $C_3 = 0$

So $$y(t) = -\frac{QE}{m\omega^2} \sin\omega t + \frac{QEt}{m\omega}$$

$$y(t) = \frac{QE}{m\omega^2}(\omega t - \sin\omega t)$$

$$\dot{z} = \frac{1}{\omega}\ddot{y} = \frac{QE}{m\omega}\sin\omega t$$

$$z = \frac{QE}{m\omega^2}(-\cos\omega t) + C$$

$z(0) = 0$

$$z(t) = \frac{QE}{m\omega^2}(1 - \cos\omega t)$$

Check units $\left[\frac{QE}{m\omega^2}\right] = \frac{[F]}{m\omega^2} = \frac{mR/t^2}{m/t^2} = R$

$$R = \frac{QE}{m\omega^2}$$

$$(y - R\omega t) = -R\sin\omega t$$

$$(z - R) = -R\cos\omega t$$

$$(y - R\omega t)^2 + (z - R)^2 = R^2$$

This is the equation of a circle that

center is at $(0, R\omega t, R)$. Motion $\perp \vec{E}$ with speed $\omega R = E/B$

Path of a spot on rim of a wheel – cycloid

Next topic <u>current</u>

need mathematical description

<u>time to get moving</u> - Last time studied force of magnetism. Today we aim to compute $\vec{B}$. $\vec{B}$ is caused by current

Current in a wire charge / time passing a given point

negative charges $\xrightarrow{\text{motion}}$ = + charges $\xleftarrow{\text{motion}}$

~~[illegible] $q\vec{v}$ as in ⊥ force~~

in wires often neg. electrons move in →
~~treated~~ + charge move in ←

$1A = C/s$ charge on electron is 1.6×10^{-19} C

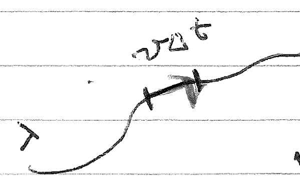
$1A = 0.625 \times 10^{19}$ elect/s

Flashlight	.5 to 1 A
Car Starter	200 A
Radio, TV	10^{-3} A =

A

Toaster 9 A

Iphone lamp



Suppose a line charge $\lambda = Q/l$ moves at a speed v ~~$I = \lambda v$~~

a length $v\,dt$ carries charge $dQ = \lambda v\,dt$

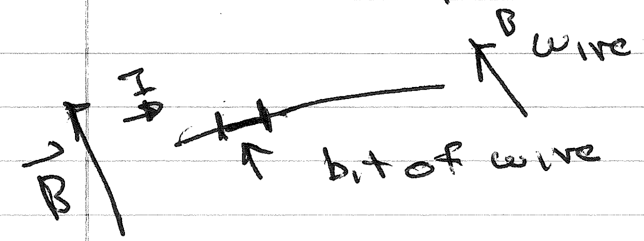
$\frac{dQ}{dt}$ = current = $\lambda v = I$

Griffiths treats current as a vector $\vec{I} = \lambda \vec{v}$

We've seen the $\vec{B}$ exerts <u>forces</u>

<u>on current carrying wire</u>

Let's compute that



carries dq

$$d\vec{F} = dq\ \vec{v} \times \vec{B}$$

note that no work is done on charges $|\vec{v}| = \text{constant}$

Sum this over wire to get total

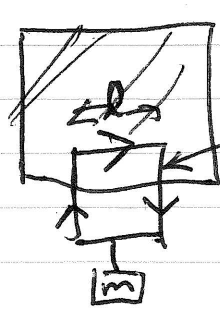
$$\vec{F} = \int dq\ \vec{v} \times \vec{B} = \int \lambda\, dl\ \vec{v} \times \vec{B}$$

$$= \int (\vec{I} \times \vec{B})\, dl$$

If $\vec{I}$ is constant along the wire $\parallel$ to direction of wire

$$\vec{F} = I \int d\vec{l} \times \vec{B}$$

Example $\vec{B}$ uniform into page

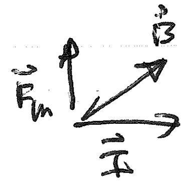


light wire with current I

What happens?

What are the forces?

gravity acts down on mass

magnetic force on wire  = up

<u>If</u> mass doesn't move

$$F_m = IB\ell = mg$$

$$I = \frac{mg}{B\ell}$$

Suppose we have this situation

Now increase the current, then mass accelerates up, Kinetic energy goes up. What does the work?

Not $\vec{B}$ (does no work)

But something must maintain current. That thing is the battery

We need a more detailed <u>description</u> <u>of currents</u>

Recall with charges we had

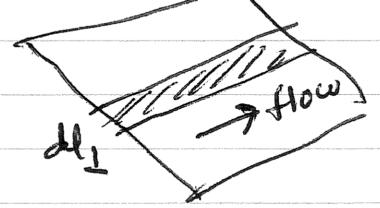
$dq = \lambda\, dl$ linear or surface

$dq = \sigma\, da$ or volume

$dq = \rho\, d\tau = \rho\, d\text{volume}$

We need similar description with charges flowing on a line - done - or on a surface, or in a volume

Suppose we have a current moving on a two-dimensional surface, Consider a ribbon of small width $dl_\perp$



$dl_\perp$ is length in direction $\perp$ to flow

Current density $= \dfrac{d\vec{I}}{dl_\perp} = \vec{K}$

K is current per unit width

If a surface charge density moves with velocity $\vec{v}$

$$\frac{dq}{dt} = \sigma v\, dl_\perp = dI$$

so $\boxed{\vec{K} = \sigma \vec{v}}$

If the surface is exposed to a $\vec{B}$

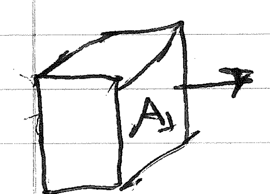
$$\vec{F} = \int (\vec{v} \times \vec{B})\, \sigma\, da = \int (\vec{K} \times \vec{B})\, da$$

If current moves in 3 dimensions

we introduce a volume current density

$$\vec{J} = \vec{I} / \text{volume}$$

charge dq = moves with $\vec{v}$



$v\Delta t$

$$dq = \rho\, A_\perp\, dl_{\parallel} \qquad \parallel \text{ to } \vec{v}$$

$$\frac{dq}{dt} = \rho\, A_\perp\, v = I$$

$$= \rho v\, A_\perp = J\, A_\perp = I$$

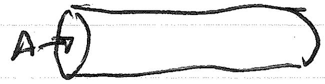
$$J = I / A_\perp \qquad A_\perp \text{ is area} \perp \text{ to current flow}$$

Total force

$$\vec{F} = \int (\vec{v} \times \vec{B})\, \rho\, d\tau$$

$$= \int (\vec{J} \times \vec{B})\, d\tau$$

Examples of currents in cylindrical wire



$A = \pi a^2$

① J = constant

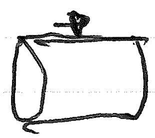
$$I = JA = \rho v A$$

② Suppose $J = ks$, s = distance from center

$$I = \int J \, da \qquad da = s\,ds\,d\phi$$

$$= \int_0^a ks \; s\,ds \int_0^{2\pi} d\phi = \frac{2\pi k a^3}{3}$$

③ Current is along edge & constant



$$I = K \, l_\perp \, v$$

$$= K \, 2\pi a \, v$$

Next step is to compute $\vec{B}$

we start with a simplification similar to electrostatics, "NOT statics"

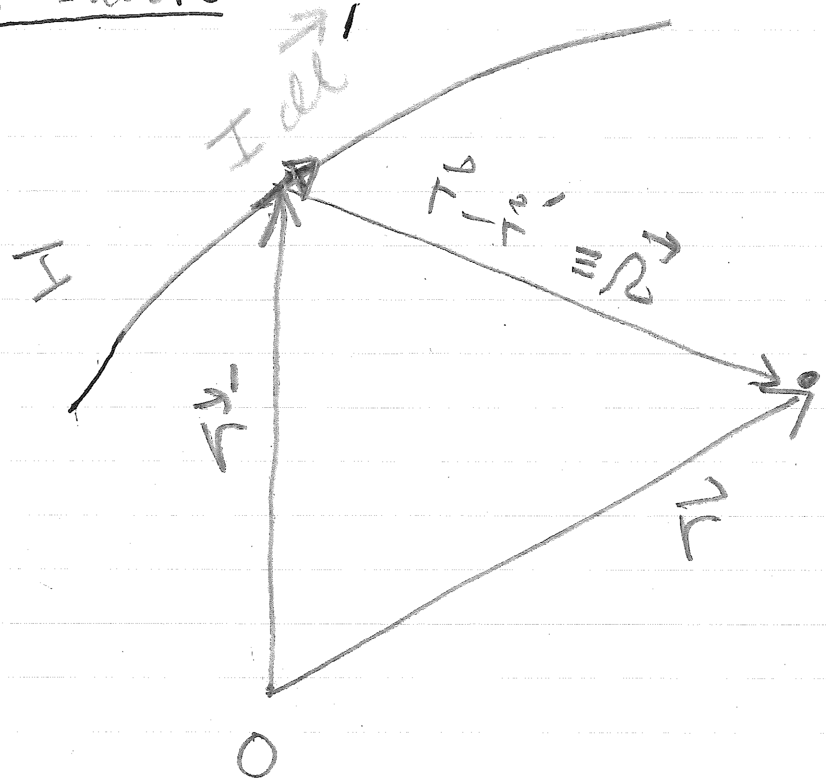
Charges will move, but no acceleration

$\frac{\partial \rho}{\partial t} = 0 \rightarrow \vec{J} = \text{constant}$

↳ denotes "steady currents"

Compute $\vec{B}$

Biot Savart Law



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I\,d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \quad (5.42)$$

or $\int \vec{K}(\vec{r}') \times (\vec{r}-\vec{r}')\,da' / |\vec{r}-\vec{r}'|^3$

$$\frac{\mu_0}{4\pi} = 10^{-7}\,\frac{N}{A^2} \qquad [B] = \frac{N}{Am} = 1\,T \text{ Tesla}$$

B of earth $\frac{1}{2}\times 10^{-4}\,T$ = 1 Gauss

inside atom $B \sim 10\,T$

largest B in lab 9/2018 1200 T U of Tokyo

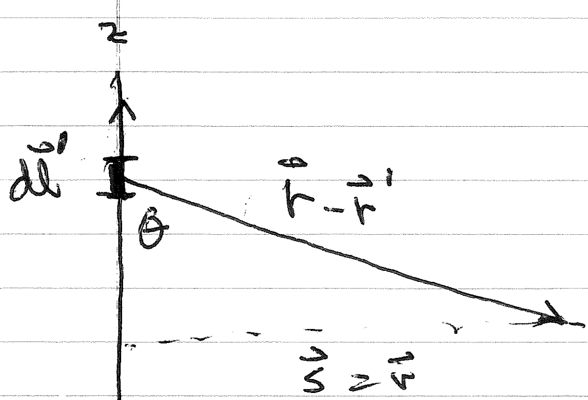
neutron star $10^8\,T$

collapsed core of star ~~having~~ before collapse $R_N \approx 10$ km
had 10–29 M_0 result from explosion gravity

Compute

$\vec{B}$ of very long straight wire

carrying I



Consider positions a distance $\vec{s}$ away from center

all such are the same for very long wire

$\vec{s}=\vec{r}$

$$\left| d\vec{l}' \times (\vec{r}-\vec{r}') \right| = dz' \, |\vec{r}-\vec{r}'| \sin\theta$$

direction of $d\vec{l} \times (\vec{r}-\vec{r}')$ is into page $= \hat{\phi}$, for all s

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \hat{\phi} I \int_{-\infty}^{\infty} dz' \frac{|\vec{r}-\vec{r}'| \sin\theta}{|\vec{r}-\vec{r}'|^3}$$

$$\sin\theta = \frac{s}{|\vec{r}-\vec{r}'|}$$

So $$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \hat{\phi} I s \int_{-\infty}^{\infty} dz' \frac{1}{|\vec{r}-\vec{r}'|^3}$$

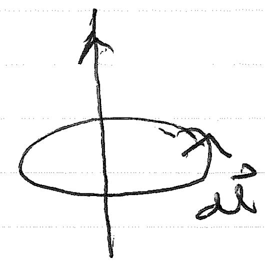
$$|\vec{r}-\vec{r}'| = \sqrt{s^2+z'^2}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \hat{\phi} I s \int_{-\infty}^{\infty} \frac{dz'}{(s^2+z'^2)^{3/2}}$$

$$\int_{-\infty}^{\infty} \frac{dz'}{(s^2+z'^2)^{3/2}} = \frac{\#}{s^2}$$ by dimensional analysis

do integral $\# = 2$

$$\boxed{\vec{B} = \frac{\mu_0}{2\pi} \frac{I}{s} \hat{\phi}}$$



Note that

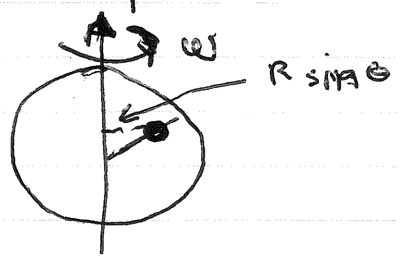
$$\oint \vec{B} \cdot d\vec{l} = B\,(2\pi s) = \mu_0 I$$

Example of Ampere's Law

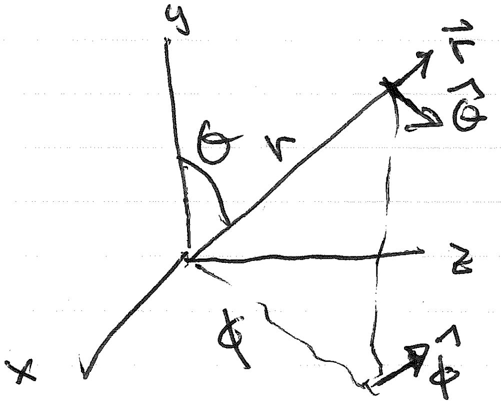
Compute $\vec{J}$

Solid, uniformly charge sphere, sphere rotates with angular frequency ω

Charge is Q, radius is R, Charge is Q



Review spherical coordinates



$\vec{r} = (r, \theta, \phi)$

what is $\vec{J}(r, \theta, \phi)$

look at bit of charge located at $\vec{r}$

$\vec{J} = \rho \vec{v}$, $\rho = \frac{Q}{\text{Volume}} = \frac{Q}{\frac{4\pi R^3}{3}} = \frac{3Q}{4\pi R^3}$

$\vec{v} = r \sin\theta \, \omega \, \hat{\phi}$

$$\boxed{\vec{J} = \frac{3Q\omega}{4\pi R^3} \hat{\phi} \, (r\sin\theta)}$$