Jan 28

Last time began accounted for magnetic ~~to~~ properties of materials. Review that

$\vec{M}$ = magnetic dipole moment / Volume

$\vec{\nabla} \times \vec{M}$ acts as a current density

This led to ~~a new~~ the result

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_f + \vec{\nabla} \times \vec{M})$$

put the curls together

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{J}_f$$

$$\vec{H} \equiv \frac{\vec{B}}{\mu_0} - \vec{M}$$

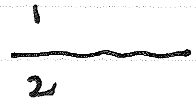
$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

Griffith $\vec{B}$ = magnetic field, $\vec{H}$ = auxiliary field or H-field

others $\vec{B}$ = magnetic induction / flux density
$\vec{H}$ = magnetic field

SI units $[B]$ = Tesla = N/Am
$[H]$ = A/m

BC

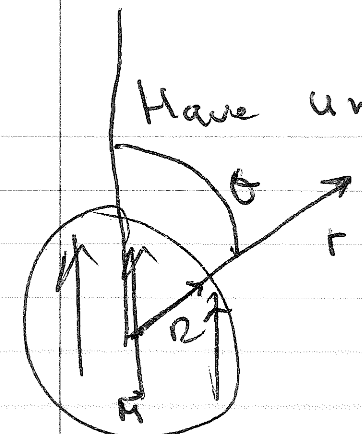


$B_{1n} = B_{2n}$ from $\nabla \cdot B = 0$

$$\vec{H}_{1t} - \vec{H}_{2t} = \vec{K}_f \times \hat{n}$$

from $\vec{\nabla} \times H \overset{\text{tangential}}{=} J_f$

Computing $\vec{B}$ & H



Have uniformly magnetized sphere

$\vec{M} = M_0 \hat{k}$ inside

outside - nothing

determine $\vec{B}$, $\vec{H}$ outside

Physics is not a spectator sport

Outside

I will do inside later

a) $\bar{d}$ $\vec{m}$

b) relate B & H

c) compute $\vec{B}$ general recall

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{3\vec{m}\cdot\hat{r}\,\hat{r}}{r^3} - \frac{\vec{m}}{r^3}\right)$$

$m = \frac{4\pi}{3} R^3 M_0$

d) compute $\vec{B}$ at $\theta = 0$, $\underset{\frac{\pi}{2}}{90^\circ}$, π

$\theta = 0$ $\quad \vec{m}\cdot\hat{r} = m$, $\quad \vec{m}\cdot\hat{r}\,\hat{r} = \vec{m}$

$\hat{r} = \hat{k}$

$$\vec{B}(\theta=0) = \frac{\mu_0}{4\pi}\,\frac{2m}{r^3}$$

$\theta = 90^\circ$ $\quad \vec{m}\cdot\hat{r} = 0$

$$\vec{B}(\theta=90^\circ) = -\frac{\mu_0}{4\pi}\,\frac{\hat{k}}{r^3}$$

$\theta = \pi$ $\quad \vec{m}\cdot\hat{r} = (-m)$, $\quad \hat{r} = -\hat{k}$

$$\vec{B}(\pi) = \vec{B}(0)$$

Do

Inside $\quad \vec{J}_f = 0$, $\quad \vec{\nabla}\times\vec{H} = 0$ $\quad \vec{\nabla}\cdot\vec{B} = 0$ gives

$$\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$$

KNOW DIV. with no $\nabla\times$ $\quad$ Sound familiar?

think electrostatics

Solve Mag as in electrostatics

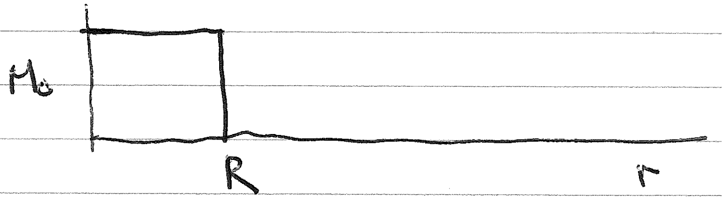
$\vec{H} = -\vec{\nabla}\Phi_M$ magnetic scalar potential

The $\vec{\nabla}\cdot\vec{H} = \boxed{-\nabla^2\Phi_M = -\vec{\nabla}\cdot\vec{M}}$ Poisson's eq for Φ_M

$-\vec{\nabla}\cdot\vec{M}$ is source of H

Now we need to workout divergence of $\vec{M}$

Look at M as a function of r



$\vec{\nabla}\cdot\vec{M} = 0$ except at surface

Need some math concepts

Theta function $\theta(x) = 1$ at $x > 0$

$= 0$ $x < 0$

$$\vec{M} = M_0\,\theta(R-r)\,\hat{k}$$

$$\vec{\nabla}\cdot\vec{M} = \frac{\partial M}{\partial z} = \frac{\partial}{\partial z} M_0\,\theta(R-r) = M_0 \left.\frac{d\theta}{dr}\right|_{r=R} \frac{\partial r}{\partial z}$$

$z = r\cos\theta$, $r = \sqrt{x^2+y^2+z^2}$

$$\frac{\partial r}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{z}{r} = \cos\theta$$

whats $\frac{d}{dr}\theta(R-r)$? $= 0$ at $r<R$
at $> R$

at $r=R$ $\frac{d\theta}{dr}(R-r) = -\infty$

Have a function $\frac{d\theta(R-r)}{dr}$ which is 0

except at 1 point $(r=R)$ then, it is

∞.

Moreover $\int_{R-\epsilon}^{R+\epsilon} \frac{d\theta(R-r)}{dr}\, dr = \theta(\epsilon) - \theta(-\epsilon)$
$= -1$

$\epsilon > 0$, $\epsilon \ll R$

A function of 1 variable that is 0 everywhere except at one point <u>and</u> which has integral = unity is called a Dirac delta function

$\delta(x) = 0$ if $x \neq 0$
$\delta(0) = \infty$

$$\int_{-\infty}^{\infty} dx\, \delta(x) = 1 \quad , \quad \int_a^b dx\, \delta(x) = 0$$

if $a > 0$
<u>or</u> if $b < 0$

$$\frac{d\theta}{dr}(R-r) = -\delta(R-r)$$

Thus $+\vec{\nabla}\cdot\vec{M} = -M_0 \cos\theta \, \delta(r-R)$

and $\nabla^2 \Phi_M = -M_0 \cos\theta \, \delta(r-R)$

The only angular dependence is $\cos\theta$

so $\Phi_m(r,\theta) = f(r)\cos\theta$

No need to make expansion in Legendre polynomial because of Fourier's thm (orthogonality)

$$\nabla^2 \Phi_M \overset{?}{=} \frac{1}{r^2}\frac{\partial}{\partial r^2} r^2 \frac{df}{dr}\cos\theta + \frac{f}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}\cos\theta$$

$$= \left(\frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}\right)^{\cos\theta} + \frac{f}{r^2}\frac{1}{\sin\theta}\frac{d}{d\theta}(-\sin^2\theta)$$

$$= \left(\frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}\right)\cos\theta - \frac{2}{r^2} f(r)\cos\theta$$

So $\frac{d^2 f}{dr^2} - \frac{2}{r}\frac{df}{dr} - \frac{2}{r^2} f = M_0 \delta(r-R)$ (1)

if $r < R$ RHS $= 0$ and $f(r) = r$ solves

if $r > R$ RHS $= 0$ and $f(r) = \frac{1}{r^2}$ solves

So function f looks like

$\Phi_M = f(r)\cos\theta \quad = f(r)\, P_1(\cos\theta)$

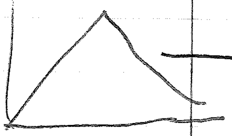
$\nabla^2 \Phi_M$ ~~$\cos\theta$~~ $= 0$ except at $r = R$

S~~olu~~

general solution of Laplace's eqn $\quad \Phi_M = A\, r \cos\theta + \frac{B}{r^2}\cos\theta$

if $r < R \qquad \Phi_M = Ar\cos\theta \quad = Ar\cos\theta$

$r > R \qquad \Phi_M = \frac{B}{r^2}\cos\theta \quad = \frac{AR^3}{r^2}\cos\theta$



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continuity at $r = R \qquad AR = B/R^2$

need only one parameter

$$\nabla^2 \Phi_M = M_0 \, \delta(r-R) \cos\theta$$

$$\int_{R-\epsilon}^{R+\epsilon} dr \; \nabla^2 \Phi_M = -M_0 \cos\theta$$

$$\nabla^2 = \frac{d^2}{dr^2} + \dots$$

The ... don't contribute as $\epsilon \to 0$ because terms are finite

So get $$\left(\frac{d}{dr} \Phi(r>R) - \frac{d}{dr} \Phi(r<R) \right) \Bigg|_{r \to R} = -M_0$$

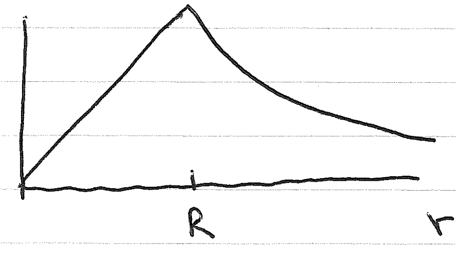
$$-\frac{2AR^3}{R^3} - A = -M_0$$

$$A = +M_0/3$$

$$\Phi_M = \frac{M_0}{3} r\cos\theta \quad , r<R$$

$$= \frac{M_0}{3} \frac{R^3}{r^2} \cos\theta \quad r>R$$

$f(r) =$



For $r<R$ $\Phi_M = \frac{M_0}{3} z$

$$\vec{H} = -\vec{\nabla} \Phi_m = -\frac{M_0}{3} \hat{k}$$

inside

$$\vec{B} = \mu_0 \vec{H} + \vec{M}\mu_0 = \frac{2}{3}\vec{M}\mu_0$$

$$\vec{B}_{outside} = \frac{\mu_0 R^3 M_0}{3}\left(\frac{3\cos\theta\,\hat{r}}{r^3} - \frac{\hat{k}}{r^3}\right)$$

outside

$$\vec{H} = -\vec{\nabla}\left[+\frac{M_0 R}{3}\frac{R^2}{r^3} r\cos\theta\right]$$

$$\left.\vec{B}_{out}\cdot\hat{r}\right|_R = \frac{\mu_0}{3} M_0 (3-1)\cos\theta$$

$$= \frac{2}{3} M_0 \mu_0 \cos\theta$$

$$= +\frac{3}{r^4}\frac{M_0 R^3}{3}\left(r\cos\theta\right)\hat{r}$$

$$- \hat{k}\frac{M_0 R^3}{3r^3} \quad = \frac{3}{r^3}R^3\cos\theta$$

$\vec{B} = \mu_0 \vec{H}$ agrees

$m = \frac{4\pi}{3} R^3 M_0$

$\vec{B}\cdot\hat{r} =$

$H_t =$ continuous because Φ is continuous

We've done electrostatics & magnetostatics. Time to get moving

Chapt 7

Current and resistance

We learned in ~~3~~ 21 that conductor in equilibrium is equipotential. $\vec{E}=0$ in conductor

However if there is a current it is not equipotential

There must be a potential difference, $\vec{E} \neq 0$ in conductor. There must be a force pushing charge. Measurements tell us that current and pot'l difference are linearly related

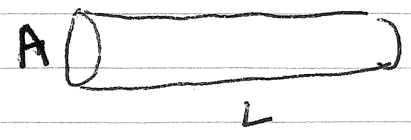
$$\underset{\text{Pot Diff}}{V} = \underset{\text{current}}{I} R \qquad \text{Ohm's Law}$$

R = resistance units in SI

$$[R] = \frac{V}{\text{amp}} = \underset{\text{Ohm}}{1\Omega} \qquad \frac{1}{R} = \text{conductance}$$

Ohm's Law is an excellent approximation for many conductors NOT UNIVERSAL

Suppose have a conducting cylinder radius R



$$R = \rho \frac{L}{A}$$

ρ → not charge density = resistivity

$$[\rho] = \Omega m$$

pure metals $\rho \approx 10^{-8}\ m\Omega$
alloys $10^{-8}\ \Omega m$

Semiconductors Ge $\sim .5\ \Omega m$
Si $2300\ \Omega m$

Insulators Wood $10^{8} - 10^{14}\ \Omega m$
Glass $10^{10} - 10^{15}\ \Omega m$

Conductivity $= \sigma = 1/\rho$ $[\sigma] = (\Omega m)^{-1}$

not charge density

So $R = \frac{L}{\sigma A}$

Insulators - energy to move electrons from bound states in atoms into states they can move is very large - QMech property

There is a more basic eqn than $V = IR$

Local form of Ohm's Law

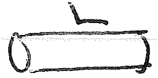
$$\vec{J}(\vec{r}) = \sigma \vec{E}(\vec{r})$$

conductivity

$\vec{J}$ is current density
$I = \int \vec{J} \cdot d\vec{a}$

low ρ → large σ → large current, again an approximation

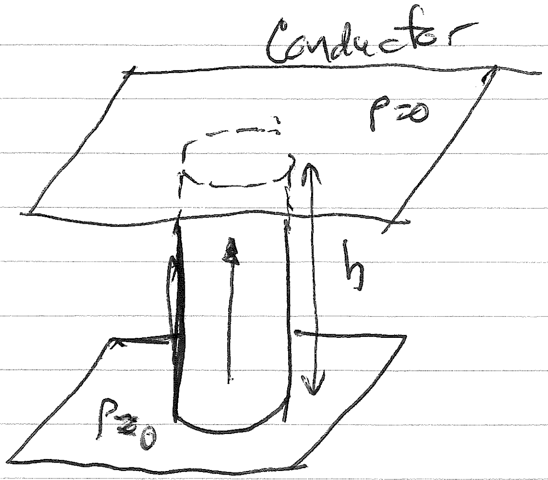
<u>Uniform</u> cylinder $I = JA$



Hlae

Last time we talked about current I
, resistance R $R = \frac{\rho L}{A}$. uniform conductor

Example cylindrical resistor between two perfect conductors.

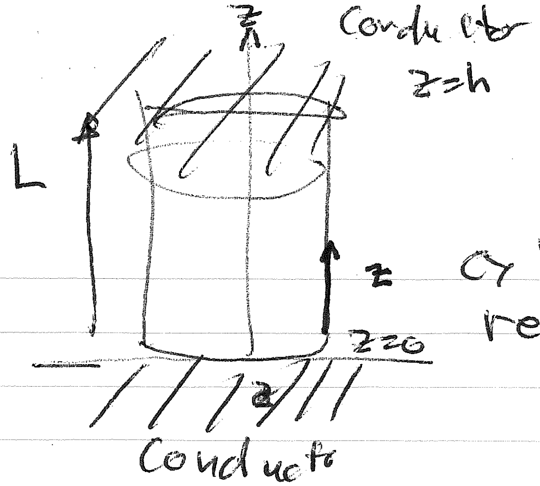


steady current I flows up. where is charge

Consider example to see where is charge?

Current is flow of charge but system is neutral

Example



Cylindrical resistor area A
height L

Above & below perfect conductor $\overset{\text{resistivity}}{\rho} = 0$

inside $\rho \neq 0$ may vary with z. $\rho(z)$

Goal compute resistance and charge distribution given a steady current I

breakup cylinder into disks of area A height dz

For a length dz we get a dR

$$dR = \frac{\rho(z)\,dz}{A}$$

Total resistance $R(z) = \frac{1}{A}\int_{z}^{h} \rho(z')\,dz'$ resistance between z and h

if ρ = constant $\rho = \rho_0$, $R = \frac{\rho_0(h-z)}{A}$

What about charge dist. Use 321 method.

to compute charge dist — get potential & use ~~Poissons eqn~~ 1st Maxwell

$$\vec{J}(\cancel{r}) = \frac{I}{A}\hat{k}$$

Pot'l diff ~~at~~ between height z and h, $V(z) = \frac{I}{A}\int_{z}^{h} dz'\rho(z')$ · Ohm's Law

from V=IR

⊙ potential diff between bottom & top causes current flow

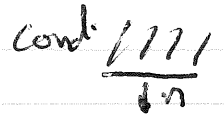
$$\vec{E} = -\vec{\nabla} V$$

$$= \frac{\rho(z) I \hat{h}}{A}$$

assume constant permittivity ϵ

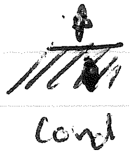
$$\rho_{free} = \epsilon \vec{\nabla} \cdot \vec{E} = \frac{\epsilon I}{A} \frac{d\rho}{dZ}$$

if $\rho = $ const get 0



σ_{free} on surface at $z = 0, h$ $\vec{\sigma} = \vec{E} \cdot \hat{n}$

$\hat{n}$ points out of conductor

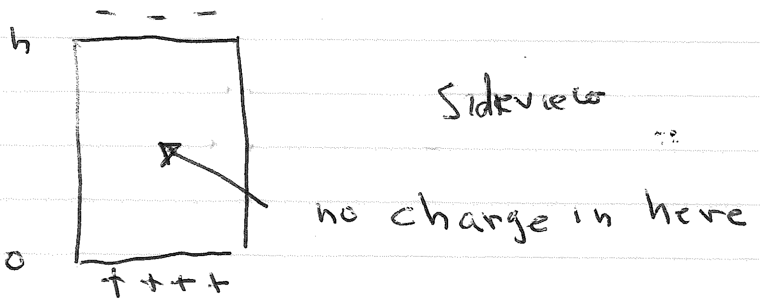


$\sigma_{free}(0)$ $\epsilon E_z(0) = \frac{\epsilon I}{A} \rho(0)$

$\sigma_{free}(h)$ $\epsilon E_z(h) = -\frac{\epsilon I}{A} \rho(h)$

If ρ is constant charges on top & bottom are = and opposite!

$\rho = $ constan



If $\rho \neq$ constant what happens?

at $z = 0$ $Q_{free} = \sigma_{free}^{(0)} A = \epsilon I \rho(0)$

$z = h$ $= -\sigma_{free}^{(h)} A = -\epsilon I \rho(h)$

In the resistor $Q_{free} = \int \rho_{free} dV = A \int_0^h dz \frac{\epsilon I}{A} \frac{d\rho}{dz}$

$= \epsilon I [\rho(h) - \rho(0))$

Sum of 3 vanishes

More on resistance

Resistance - an act of resisting
opposition

In metals

Ohm's Law $\vec{J} = \rho\vec{v} \propto \vec{E} \propto \vec{F}$

$\vec{v} \propto \vec{F}$ is Newton wrong?

No. There is opposition in metals El. forces act on electrons

they have to move through + posit recharge & other electrons
atoms in lattice

Basically in conductor conducting electrons

lose energy by collisions with atoms

Energy lost is transferred as heat in the

form of kinetic energy of atoms. Power is

lost. How much??

The work/per unit time as charge passes

through pot'l diff V is is $\frac{dq}{dt}V = IV$ $[P] =$ Watts
~~eo~~ charge/time V =

Energy conservation tells us that this power

is dissipated as heat power $P = IV = I^2R$

Joule's law 1841 found by

Experimental measurement

	Current A	Power W
Table Lamp	0.8	100
Hand Iron	8	1000
Starter motor of car	180	2200

R differs in these devices

Show demos

5K 10. Induced currents o forces

10.15 wire o mag

10.18 Coil Pendulum in magnet

10.20 Coil o bar mag

10,30/40 Coil "

Mutual inductor

It's time for time dependence

Thursday Jan 24

5K: Electromagnetic Induction

5K10. Induced Currents and Forces

Wire and Magnet (5K10.15) -- A single loop of wire is passed between poles of a large horseshoe magnet, causing current to flow (shown on a galvanometer). The faster the wire is moved, or the greater the number of loops, the larger the current.

Current moving in mag field

Coil Pendulum in Magnet (5K10.18) -- A pendulum with a large coil for a bob swings between the poles of a large horseshoe magnet. A small light bulb wired to the coil flashes when the coil swings through the magnetic field. Time dependent B cause I

Simple Coil and Bar Magnet (5K10.20) -- A coil is connected to a galvanometer. A bar magnet is passed through the coil and the galvanometer measures the current. Time dep B

10/20/40 Turn Coils with Magnet (5K10.21) -- Coils of 10, 20, and 40 turns wired in series on a common stand, through which a permanent magnet is moved to produce small currents as shown on a galvanometer.

Mutual Induction (5K10.30) -- Two coils slide on a track so that the distance between them can be varied. Current is pulsed into one coil with a switch, which induces current in the second coil. Meters show the currents in both coils, and show that there must be a changing current in the first coil to induce current in the second. Intensity of induced current changes with separation, and various metal cores can be inserted to determine their effect on the magnetic flux.

All of the effects shown in the demos

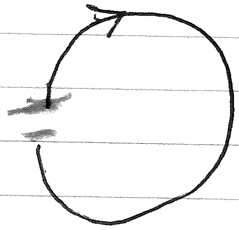
Can be explained in one basic equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ Faraday's Law

This eqn is the result of many experiments

We'll spend time explaining it using historical approach

Consider a steady current in a complete circuit



In electrostatics If a charge q goes around a complete circuit $\Delta V = 0$, no change in PE.

But in electric circuit charge loses PE as it moves $V = IR$. This means that with steady current in complete circuit something must supply energy. Somewhere PE must increase

An electric circuit is [illegible] analogous to
Drumheller fountain
an ornamental water fountain that recycles its water. Water pours out of openings at the top, cascades down and collects in a basin at bottom. A pump lifts water to top. This is in increase in pot'l energy. W.O pump H_2O just stays down

In any electric circuit must be something that acts like water pump. Something that makes ~~w~~ charge go from lower to higher pot'l. This influence is called <u>electromotive</u> = <u>force</u> = emf $\mathcal{E}$ = Work / unit charge

Any complete circuit uses device that supplies emf.

Examples

Battery
electrical generators
Solar cells
thermo couples
fuel cells

converts mechanical, chemical, thermal, energy into electr energy.

Define $\mathcal{E}$ by potential diff around closed circ.

$$\mathcal{E} = +\oint \frac{\vec{F}}{q} \cdot d\vec{l} \neq 0$$ Something new

$\vec{F}$ = force

$\vec{F}$ has two sources $= q(\vec{E} + \vec{v} \times \vec{B})$ ordinary $\vec{E}$ + extra stuff only extra stuff contributes to total line integral

=

Anew way to generate EMF:
Electromagnetic induction : time changing magnetic flux $\longrightarrow \vec{E} \rightarrow I$

Electromagnetic induction is the basis of ~~all~~ many machines we use in daily life

We have seen in the demos effects of the $q\vec{v}\times\vec{B}$ term of $\vec{F}$ - motional EMF caused by Motion of circuit in magnetic field

But we need more explanation of $q\vec{E}$

$\vec{E}$ has two sources electrostatic due to charges at rest and a new effect coming from $\frac{\partial\vec{B}}{\partial t}$. The electrostatic field causes no EMF because $(\nabla\times\vec{E})_{static}=0$

$$\oint \vec{E}\cdot d\vec{l} = \int(\vec{\nabla}\times\vec{E})\cdot d\vec{a} = 0$$

However there is a new term inductive which cause $\vec{\nabla}\times\vec{E}\neq 0$

In fact we will see $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$

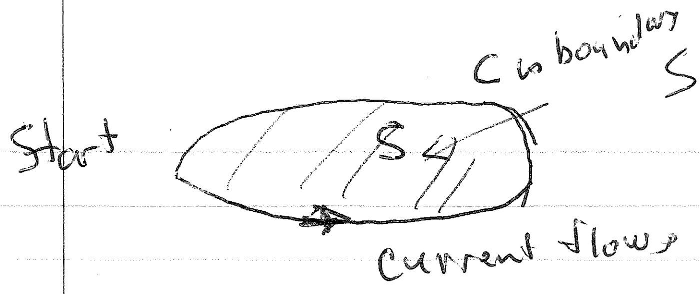
So two kinds of EMF 1) circuit moving in magnetic field 2) $\partial\vec{B}/\partial t \neq 0$

1) $\vec{B}(\vec{x})$ no time dependence wire moves with velocity $\vec{v}$

Goal compute

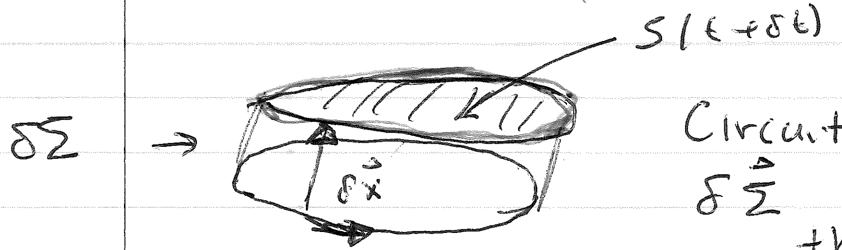
$$\mathcal{E} = \oint_C (\vec{v}\times\vec{B})\cdot d\vec{l} = \oint_C \vec{B}\cdot(d\vec{l}\times\vec{v})$$

Surface S(t) moves



This rigid wire loop moves in a magnetic field

after time δt, (δt small) we have



Circuit sweeps out a surface $\delta\vec{\Sigma}$ shaped like a ribbon the sides are the ribbon

$\mathcal{E}$ needs

$$d\vec{l}\times\vec{v} = d\vec{l}\times\frac{\delta\vec{x}}{\delta t}$$

but $d\vec{l}\times d\vec{x}$ = outward directed area element $=\delta\vec{\Sigma}$

side of cylinder ribbon

Thus $\oint_c \vec{B}\cdot(d\vec{l}\times\vec{v}) = \oint \frac{\vec{B}\cdot d\vec{\Sigma}}{\delta t}$,

$\mathcal{E} = \frac{\delta\Phi_r}{\delta t}$ Φ_r is flux of $\vec{B}$ through ribbon

Now consider closed surface Made of

$\delta\Sigma$ and top $S(t+\delta t)$ & bottom $S(t)$

$\oint \vec{B}\cdot d\vec{a}=0$ closed surface because $\vec{\nabla}\cdot\vec{B}=0$

since $\vec{\nabla}\cdot\vec{B}=0$ flux through closed surface $=0$

$$\delta\Phi_r + \left(\int_{S(t+\delta t)} \vec{B}\cdot d\vec{a} - \int_{S(t)} \vec{B}\cdot d\vec{a}\right) = 0$$

$$\mathcal{E} = \frac{\delta\Phi_r}{\delta t} = -\frac{1}{\delta t}\left(\qquad\right) = -\frac{d}{dt}\Phi(t)$$

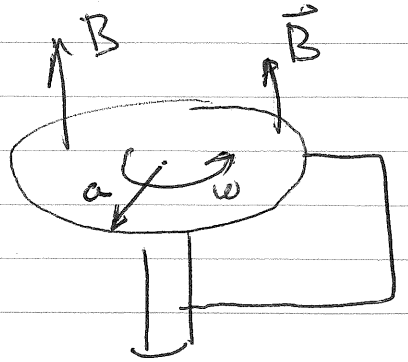
flux through surface S that bounds C

Last time we saw how time dependence causes forces that cause currents $\mathcal{E} = -\frac{d\Phi}{dt}$

Start with a couple of examples

Show Faraday disk generator

Cu Disk spins in uniform magnetic field $\vec{B} = B\hat{z}$ 5K40.15



whats going on free charges rotate

have velocity – Lorentz force ~~of~~ unit charge

$$\vec{v} \times \vec{B} = vB\,\hat{\phi}\times\hat{z} = vB\,\hat{s} = \omega s B\,\hat{s}$$

gets EMF $= \mathcal{E} = \int_0^a \omega s B\, ds = \frac{\omega a^2}{2}$

$I = \mathcal{E}/R$

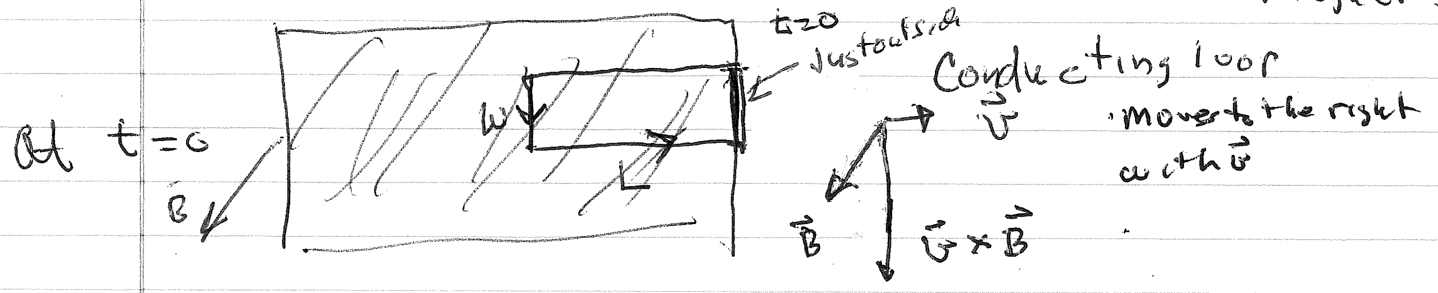
Can't compute from the $\mathcal{E} = -\frac{d\Phi}{dt}$

because ~~flux~~ this assumes current is in well-defined path, Here current spreads radially

The net result $\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$

Simple example

B points out of page in shaded region (between poles of magnet)



The motional EMF is

$$\mathcal{E} = \oint (\vec{v}\times\vec{B})\cdot d\vec{l}$$

at $t=0$ $\mathcal{E} = vBW$

left end is in $\vec{B}$, top & bottom cancel

consider Compute $\frac{d\Phi}{dt}(0)$

time dependence: length between pole faces (in $\vec{B} \neq 0$) is $L-vt$

Flux is $\Phi = B(L-vt)W$

$$\frac{d\Phi}{dt} = -BvW = -\mathcal{E}$$

$$\mathcal{E} = BVW$$

Direction of induced current down

RH rule says a new B is generated by the induced current

This $\vec{B}$ is in the direction <u>into</u> the page <u>opposing</u> the original $\vec{B}$

The next step is to derive

$$\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$$

The eqn $\mathcal{E} = -\frac{d\Phi}{dt}$ is integral

~~eqn~~ relation, we can get a local relation

between derivatives of $\vec{E}(\vec{r},t)$, $\vec{B}(\vec{r},t)$ using Stokes thm

$\mathcal{E}$ is a potential difference per unit charge

LHS $\mathcal{E} = \oint_C \vec{E}\cdot d\vec{l} = \int_S d\vec{a}\cdot(\vec{\nabla}\times\vec{E})$ suppose S is fixed

RHS $= -\frac{d\Phi}{dt} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$

$$\oint_S d\vec{a}\cdot\left(\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t}\right) = 0$$

True for all surfaces everywhere & all times

So $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ $\vec{E}$ is induced

Summarizes all ways time dep B causes E

can ~~have~~ change B by changing currents, moving loop, move source of B

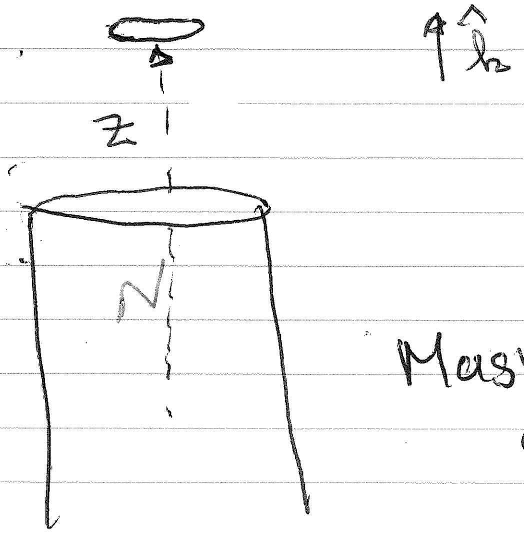
Sign is −. This guarantees that

the currents caused by the induced $\vec{E}$ oppose the original magnetic field.

There is no free lunch! → Lenz's Law

Example

Physsis is not a speed. sport

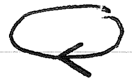


Magnet $\vec{B}$ points $\hat{k}$ | $\hat{z}$
up.
Fills area of Loop

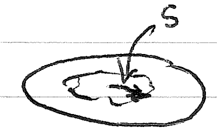
have circular loop and drop it falls without rotation

what happens

What is direction of induced current ?



Now suppose take view of circuit



$\vec{B}(t)$ fills the area $\vec{B}$ indep of $\vec{x}$ $\vec{s}$ (distance from center of loop at fixed z) and direction is up

Next

Determine the induced electric field

Symmetry is cylindrical Direction of $\vec{E}$ must be $-\hat{\phi}$ curling around $-\frac{\partial \vec{B}}{\partial t}$

at t=0 $\vec{E} = -E_\phi(s)\,\hat{\phi}$ $E_\phi(s) = |\vec{E}| > 0$

A r way does not use loops

$$\vec{\nabla} \times \vec{E} = -\frac{1}{s} \frac{\partial}{\partial s} (s E_\phi) \hat{k} = -\frac{dB}{dt} \hat{k}$$

$$\frac{d}{ds} s E_\phi = \frac{dB}{dt} s$$

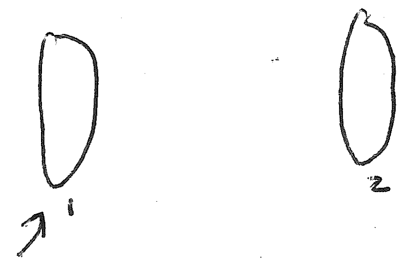
$$s E_\phi = \frac{dB}{dt} \frac{s^2}{2}$$

$$\underline{E_\phi = \dot{B} \frac{s}{2}}$$

Inductance

Currentin a circuit cause B and flux thru anotha circuit (or the same circuit). If current changes with time in first circuit, current induced in second or same circuit. name is Inductance, Let's see what we can say about inductance

We saw a demo in which



time changing B caused a current in 2

A current is induced the name is mutual inductance,

Suppose now a steady current in ①, I_1, causes a $\vec{B}_1$ due to wire 1
There will be a flux thru 2.

$$\Phi_2 = \int \vec{B}_1 \cdot da_2 \propto I_1 \quad \text{(Biot Savart)}$$

$$\Phi_2 = M_{21} I_1$$

Calculations of M_{21} may be complicated doing 1 integral to get $\vec{B}_1$ and another to get the flux ~~in~~ thru 2.

There is a nice formula that allows us to see some general features. The trick is to use the vector potential. Use $\vec{B}_1 = \vec{\nabla}\times\vec{A}$ in Φ_2:

$$\Phi_2 = \int_S (\vec{\nabla}\times\vec{A}_1)\cdot d\vec{a}_2$$

Stoke's thm

$$= \oint_C \vec{A}_1\cdot d\vec{l}_2$$

C is the boundary of S

But $$\vec{A}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\vec{l}_1}{\mathscr{r}}$$

so $$\Phi_2 = \underbrace{\left[\frac{\mu_0}{4\pi}\oint\oint\frac{d\vec{l}_1\cdot d\vec{l}_2}{\mathscr{r}}\right]}_{\equiv\, M_{21}} I_1$$

Neumann formula

Double line integral:

M_{21} is purely geometrical - depends on sizes, shapes & positions

The integral is unchanged if we switch the subscripts 1 & 2. For any two loops flux thru 1 due to current in 2 is the same as the flux thru loop 2 due to same current in 1