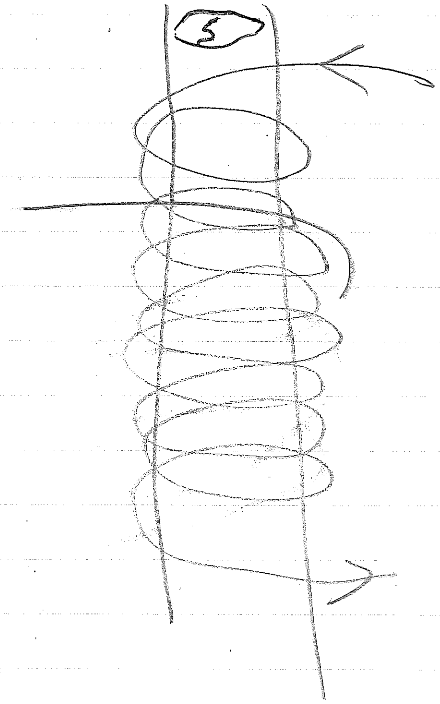
7.3

Transformer

Transformer demo

5K30.20



$\mathcal{E}_2$

Solenoid

C_1 N_1 turns, length l_1

2 100 sets bound

cross sect
$S =$ area

Φ_{21} B_1 = field of solenoid

$= \frac{\mu_0 N_1 I_1}{l_1}$

$\Phi_{21} = N_2 S B_1$

$M_{21} = \frac{\Phi_{21}}{I} = \frac{\mu_0 N_1 N_2 S}{l_1}$

L_1 = self inductance

$\mathcal{E}_2 = -\frac{d\Phi_{21}}{dt} = -M_{21}\frac{dI_1}{dt}$

$= \mu_0 N_1 N_2 S/l_1 \frac{dI_1}{dt}$

what about coil 1? there is an emf induced thru its own circuit. This is called self inductance

$\mathcal{E}_1 = \frac{\mu_0 N_1^2 S}{l_1}\frac{dI_1}{dt}$, $\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$

Mutual inductance is the principle behind

<u>transformer</u>. [illegible] coil [illegible]

to chan[illegible] coil

An AC transformer may increase (step up)

or decreases voltage from primary to

secondary. Ignition in car takes

input voltage alternating between 0 and 12V

and steps it up to high output voltage so that

$\vec{E}$ at spark plug > breakdown

field for the air ~~fuel~~ mixture in cylinder,

spark is produced, mixture ignites

Transformers - history of AC; electric

power → LD ch[illegible] transm. higher V) use

longdistan trans mission line

than household

Development of transformers was an important step

in the history of AC ~~development~~ electri power. Long dst transmission

lines use much higher voltage than household

appliances & electric lights. Transformers can be

seen on telephone poles

The calculation of $\mathcal{E}_1$ is an example of self inductance. If there is a current in a loop There will a B and therefore a flux. The flux is propto current

so we write $\Phi = LI$. For earlier example $L = \frac{\mu_0 N^2 S}{l}$

L is the self inductance. If I change with t

$$\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

L is useful to calculate energy in $\boxed{\text{magnetic}}$ $\boxed{\text{field}}$. B carries energy because we must do work to create the currents and keep them going

power that must be supplied to maintain a current I in a circuit, because the induced EMF $= \mathcal{E} - \quad -L\frac{dI}{dt}$

$$P = \frac{dW}{dt} = \quad \text{charge per unit time} \times \text{Voltage diff}$$

$$= -I\mathcal{E}$$

So $= \frac{dW}{dt} = -I\mathcal{E} = + \quad I \, \cancel{d} \frac{dI}{dt}$

The total energy is $\int \frac{dW}{dt}\,dt = \frac{1}{2} L I^2$

energy in B field is

$$\boxed{W = \frac{1}{2} L I^2}$$

We want an expression ~~for~~ which has more details - $\vec{B}(\vec{r})$ varies with space want energy density

For Electric fields
the stored energy density,

energy per unit volume is $\frac{\epsilon E^2}{2}$

need expresion for magnetic contribution

This is important for understanding

energy & momentum carried by a

beam of light.

Last Time energy in magnetic field

$W = \frac{1}{2} L I^2$ energy required to make magnetic field

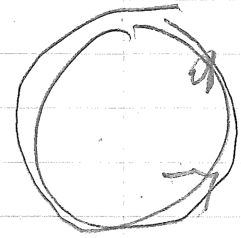
We want to get ~~an~~ expression in terms of $\vec{B}(\vec{r},t)$. Why? Recall

energy density of ~~~~ electric field is $\frac{1}{2} \epsilon E^2$, $\vec{E}$ & $\vec{B}$ are two sides of same coin. Want theory electromagnetism

When computing energy need both

energy flows between electric & magnetic write in terms of B. Want B^2 (why)

$$\Phi = LI = \int \vec{B}\cdot d\vec{a}$$



I

$$= \int (\vec{\nabla}\times\vec{A})\cdot d\vec{S} = \int \vec{A}\cdot d\vec{l}$$

use $W = \frac{1}{2}LI^2 = \frac{1}{2}\Phi I$

Now want more general: a set of current loops

$$W = \frac{1}{2}\int \vec{A}\cdot\left(I\,d\vec{l}\right)$$

current density

$$I\,d\vec{l} = \vec{J}\,d\tau$$ ← volume element

$$W = \frac{1}{2}\int \vec{A}\cdot\vec{J}\,d\tau$$

enters in field theory

integral over region of current

$$\vec{J} = \frac{1}{\mu_0}\vec{\nabla}\times\vec{B}$$

$\boxed{\text{magnetostatics}}$

$$W = \frac{1}{2\mu_0}\int \vec{A}\cdot(\vec{\nabla}\times\vec{B})\,d\tau$$

Almost looks like B^2. Int by parts

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) \boxed{- \vec{A}\cdot(\vec{\nabla}\times\vec{B})}$$

$$W = \frac{1}{2\mu_0}\int d\tau\left[\vec{B}\cdot\vec{B} - \vec{\nabla}\cdot(\vec{A}\times\vec{B})\right]$$

$$= \frac{1}{2\mu_0}\int_V d\tau\,\vec{B}\cdot\vec{B} - \int_S d\vec{a}\cdot(\vec{A}\times\vec{B})$$

S = boundary of V

"Integration over all space

$S \to \infty$

$d\vec{a} \to r^2 d\Omega\, \hat{n}$

Magnetostatics
part of story

$B \sim \frac{1}{r^3}$ $A \sim \frac{1}{r^2}$

$$\boxed{W_M = \frac{1}{2\mu_0}\int d\tau\, B^2}$$

dens $= \frac{B^2}{2\mu_0}$

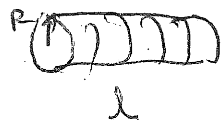
Elect E $\frac{\varepsilon_0 E^2}{2}$

dens

This is not the whole story
an important element is missing -
with this argument so far
light waves would not propagate

Check out formula
a simple example

W of long solenoid



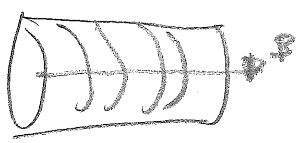
$$W = \frac{1}{2} L I^2$$

$$= \frac{1}{2} \mu_0 \frac{N^2}{l} \pi R^2 I^2$$

$$L = \mu_0 \frac{N^2 \pi R^2}{l}$$

get from $W = \frac{1}{2\mu_0} \int B^2 \, d\tau$

$$B = \mu_0 \frac{N}{l} I$$



$$\int d\tau = \pi R^2 l$$

$$W = \frac{1}{2\mu_0} B^2 \int d\tau$$

$$= \frac{1}{2\mu_0} B^2 \pi R^2 l$$

$$= \frac{1}{2} \mu_0 \frac{N^2}{l} I^2 \pi R^2$$

Same result

before

As of $\leq$ Maxwell < 1861 Basic form

Coulomb $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$ $\rho = \rho_b + \rho_f$

Exp't $\vec{\nabla} \cdot \vec{B} = 0$

Faraday $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Ampere $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ $\vec{J} = \vec{J}_b + \vec{J}_f$

Extra conservation of charge $\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

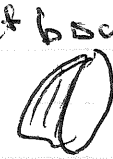
LFL $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

There are some problems with these
$\frac{\partial \vec{B}}{\partial t}$ causes $\vec{E}$
But $\frac{\partial \vec{E}}{\partial t}$ does not appear
$\vec{E}$ & $\vec{B}$ are related
another problem involves charge

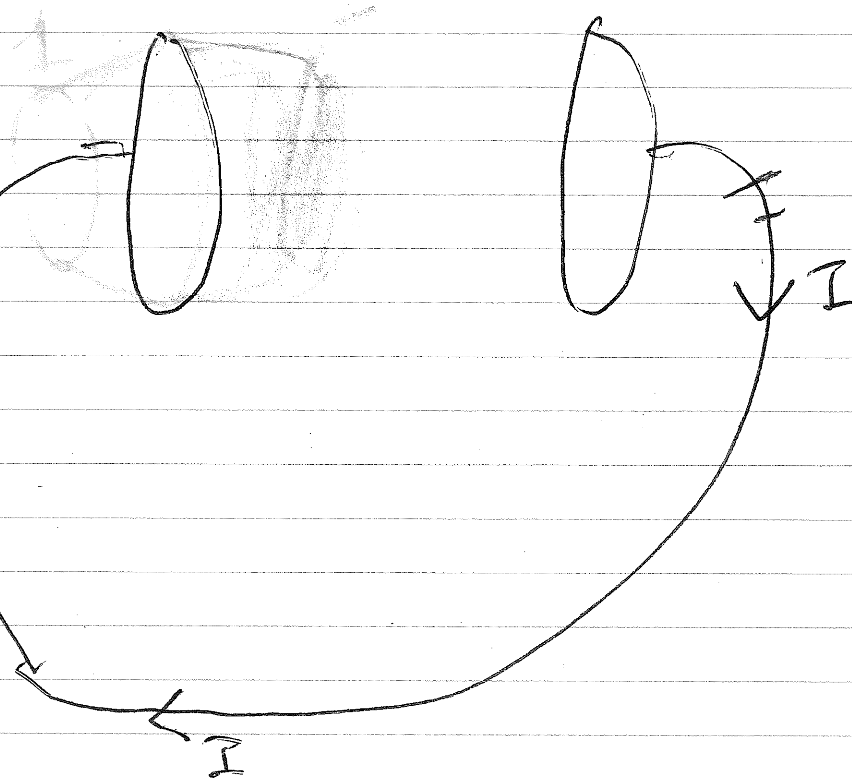
Ampere's can be written as
$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \int_a \vec{J} \cdot d\vec{a}$$

But there are an infinite # of possible surfaces a that bound c

or



Capacitor charging



① $\int_S \vec{J} \cdot d\vec{a} = I$ ② $\int \vec{J} \cdot d\vec{a} = 0$

Inconsistency Further more math

can take $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 \vec{\nabla} \cdot \vec{J}$

$0 = \vec{\nabla} \cdot \vec{J}$ only true for steady current

but $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ inconsistency

Maxwell add a term (not historically accurate)

that maintained charge conservation

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \vec{X}$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0 = \mu_0 \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{X}$$

$$\vec{\nabla} \cdot \vec{X} = \mu_0 \frac{\partial \rho}{\partial t}$$

Look at Coulomb
$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$

$$\vec{X} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{X} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{E} = \mu_0 \epsilon_0 \frac{\partial \rho}{\partial t \, \epsilon_0}$$

So Maxwell's eq changes Ampere to

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$\vec{J}_D$ = displacement current $= \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \vec{J}_D) \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Now consider previous example

area ① $\int \vec{J} \cdot d\vec{a} = I$

② $\int J_D \, d\vec{a} = \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}$

$I_{enclosed} = 0$
but J_D exists

$$\Phi_E = EA = \frac{\sigma}{\epsilon_0} A$$

$$\frac{d\Phi_E}{dt} = \frac{d\sigma}{dt} \frac{A}{\epsilon_0} = \frac{dQ}{dt} \frac{1}{\epsilon_0}$$

$$\int \vec{J}_D \cdot d\vec{a} = \epsilon_0 \, dQ/dt / \epsilon_0 = dQ/dt = I$$

Now

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0 \qquad \rho = \rho_b + \rho_f$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{J} = \vec{J}_b + \vec{J}_f$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

These are complete

Magnetic monopoles

In general $\vec{E}$ & $\vec{B}$ related

what is charge in one ref frame is

current in another. So $\vec{E}$ is transformed into $\vec{B}$

If we look at Maxwell in case $\rho = 0, \vec{J} = 0$

have $\vec{\nabla} \cdot \vec{E} = 0$ $\quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \cdot \vec{B} = 0$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

equations invariant under transform

$\vec{E} \to \vec{B}$

$\vec{B} \to -\mu_0 \epsilon_0 \vec{E}$

$\exists$ symmetry between $\vec{E}, \vec{B}$ if $\rho, \vec{J} = 0$

but with sources $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0 \quad \vec{\nabla} \cdot \vec{B} = 0$

major difference between $\vec{E}$ & $\vec{B}$

We can make a more symmetric

version of Maxwell's eqns

$$\vec{\nabla}\cdot\vec{E} = \rho_e/\epsilon_0 \qquad \vec{\nabla}\times\vec{E} = -\mu_0 \vec{J}_m - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = \rho_m \mu_0 \qquad \vec{\nabla}\times\vec{B} = \mu_0 \vec{J}_e + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

put in mag. charge density - it must obey continuity

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}\cdot\vec{J}_m = 0 \quad , \quad \vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) \text{ must vanish}$$

This possibility has been very attractive to physicists. There have been at least 1400 papers with title magnetic monopoles

physicists have searched here there & everywhere - North pole south pole

Outer space cosmic rays moon rocks

important in grand unified theory

predicted too many monopoles

inflationary universe

~~Will talk a'~~ one monopole was found but it went away

PRL 48, 1378

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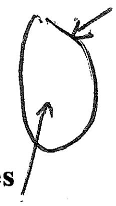
superconduct ring
magn. flux create E & current



First Results from a Superconductive Detector for Moving Magnetic Monopoles

S_Γ

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A velocity- and mass-independent search for moving magnetic monopoles is being performed by continuously monitoring the current in a 20-cm^2-area superconducting loop. A single candidate event, consistent with one Dirac unit of magnetic charge, has been detected during five runs totaling 151 days. These data set an upper limit of 6.1×10^{-10} cm^{-2} sec^{-1} sr^{-1} for magnetically charged particles moving through the earth's surface.

PACS numbers: 14.80.Hv

$\mathcal{E} = IR$

The detection of a moving magnetic charge with a superconducting ring is based solely on the long-range electromagnetic interactions between the magnetic charge and the macroscopic quantum state of the ring.[1] Such a detector measures a moving particle's magnetic charge regardless of its velocity, mass, electric charge, or magnetic dipole moment. In this paper, the first experimental results from use of this scheme are presented.

Superconductors make natural magnetic charge detectors, as suggested by comparing the flux quantum of superconductivity $\varphi_0=hc/2e$ with the flux emanating from a single Dirac charge $4\pi g=hc/e$. Dirac[2] was led to his value for the elementary magnetic charge by postulating that the wave function of a single electron in the field of a pole should be single valued. In superconductivity, the postulate of a single-valued macroscopic wave function leads to flux quantization. The factor of 2 arises from the electric charge, $2e$, of the Cooper pairs.

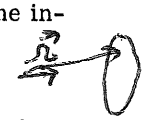
Consider a magnetic charge g moving at velocity v along the axis of a superconducting wire ring of radius b. Integrating Maxwell's generalized equation for the monopole current, $\mathrm{curl}(\vec{E})+(1/c)d\vec{B}/dt=-(4\pi/c)\vec{j}_m$, over the area S_Γ in the plane of the ring, we obtain

$$\oint_\Gamma \vec{E}\cdot d\vec{l}+c^{-1}(d\varphi/dt)=-(4\pi g/c)\delta(t), \qquad (1)$$

where S_Γ is bounded by a path Γ that is everywhere inside the wire. If we neglect the finite response time of the superelectrons, $\vec{E}$ will vanish along Γ and $\varphi(t)=-4\pi g\theta(t)$, where we set $\varphi=0$ for $t=-\infty$. The total flux through S_Γ is $\varphi=\varphi_g+\varphi_s$, φ_g from the monopole and φ_s from the induced supercurrent. We find

$R=0$

integrate (1)



$$\varphi_g(t)=2\pi g\left[1-2\theta(t)+\frac{\gamma vt}{[(\gamma vt)^2+b^2]^{1/2}}\right],$$

$\vec{B} = \frac{g}{4\pi r^2}\hat{\Omega}$

and $\varphi_s=-I(t)L$, where $I(t)$ is the induced supercurrent and L the self-inductance of the ring. Thus, substituting $4\pi g=2\varphi_0$, we obtain

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = 1$

$$I(t)=\frac{\varphi_0}{L}\left[1+\frac{\gamma vt}{[(\gamma vt)^2+b^2]^{1/2}}\right].$$

This result is independent of the choice of surface S_Γ bounded by the path Γ and corresponds to a change of $2\varphi_0$ through the ring (Fig. 1). The change in current will occur with a characteristic time given by $b/\gamma v$.

In the general case, any trajectory of a magnetic charge g which passes through the ring will result in a flux-quanta change of 2, while one which misses the ring will produce no flux change. In the less likely event that a magnetic charge passes through the ring wire itself, it will leave a trapped doubly quantized vortex, and some intermediate total current will persist. Any electric charge or magnetic dipole moment of the particle

$\vec{J}_m = g\hat{k}v\,\delta(z-vt)\,\delta(x)\delta(y)$ need at $z=0$

$= g\hat{k}\,\delta(t)\,\delta(x)\delta(y)$

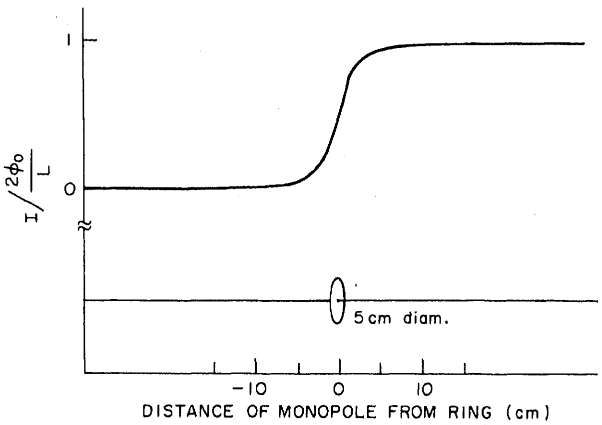


FIG. 1. Induced current in a superconducting ring for an axial monopole trajectory.

will cause only small transient fluctuations and no dc shifts. Thus, moving magnetically charged particles can be detected by monitoring the current in the superconducting ring.

In this experiment a four-turn, 5-cm-diam loop, positioned with its axis vertical, is connected to the superconducting input coil of a SQUID (superconducting quantum interference device) magnetometer.[3] The passage of a single Dirac charge through the loop would result in an $8\varphi_0$ change in the flux through the superconducting circuit, comprised of the detection loop and the SQUID input coil (a factor of 2 from $4\pi g = 2\varphi_0$ and of 4 from the turns in the pickup loop). The SQUID and loop are inside a 20-cm-diam, 1-m-long cylindrical superconducting shield closed at the bottom, and these are mounted inside a single Mumetal cylinder. The combined shielding provides 180-dB isolation from external magnetic field changes and an ambient field of 5×10^{-8} G.[4]

The voltage output of the SQUID electronics, which is directly proportional to the supercurrent in the detection loop, is continuously recorded through a 0.1-Hz low-pass filter onto a strip-chart recorder. In addition, several times per day digital voltmeter readings are taken to guard against recorder failures.

The detector sensitivity has been calibrated in three independent ways: (a) by measuring the SQUID response to a known current in calibration Helmholtz coils and calculating their mutual inductance to the superconducting loop (± 4%); (b) by estimating the self-inductance of the superconducting circuit (± 30%); and (c) by directly observing flux quantization within the superconducting circuit (± 10%). All three methods agree within their independent uncertainties.

Two additional effects influence the exact detector response. First, a magnetic monopole whose trajectory intersects either the transformer loop in the SQUID, the twisted leads from the SQUID to the loop, or the loop wire itself would produce a shift of nonintegral magnitude. Computation of the average area ratio of the loop to the remainder of the transformer circuit indicates that such events will be suppressed by a factor of 25 compared to loop events. Second, a particle traversing the superconducting shield will leave doubly quantized vortices wherever the trajectory intersects a wall. The effect is a magnetic field change inside the shield and an applied flux change across the loop. The total induced current change in the loop is $\Delta I = (8\varphi_0/L)[\eta - \xi(A_l/A_S)]$, where $A_l/A_S = 0.06$ is the ratio of the loop to shield cross-sectional areas. For a trajectory that intersects the loop, $\eta = 1$, and for one that misses, $\eta = 0$. The geometric factor ξ depends on the trajectory impact parameter and inclination angles, and has maximum value of 1 for axial trajectories through the shield and a minimum of 0 for transverse ones. Current changes of $(0.06)8\varphi_0/L$ or less will be observed for trajectories that pass through the shield but not the loop. The probability for such events with $\Delta I > (0.02)8\varphi_0/L$ is about 10 times larger than for the loop.

As of 11 March 1982 data have been recorded for a total of 151 days. Several intervals throughout a continous one-month time period are shown in Fig. 2(a), where no adjustment of the dc level has been made. Typical disturbances caused by daily liquid-nitrogen and weekly liquid-helium transfers are evident. A single large event was recorded [Fig. 2(b)]. It is consistent with the passage of a single Dirac charge within a combined uncertainty of ± 5% (resulting from the calibration uncertainty and the distribution of geometric factor ξ). It is the largest event of any kind in the record. In Fig. 3 are plotted the 27 events exceeding a threshold of $0.2\varphi_0$, which remain after exclusion of known disturbances such as transfers of liquid helium and nitrogen.[5] An event is defined as a sharp offset with well-defined stable levels for at least 1 h before and after. Only six events were recorded during the 70% of the running time when the laboratory was unoccupied.

The following statements about spurious detector response can be made:

(a) *Line voltage fluctuations* caused by two power outages and their accompanying transients

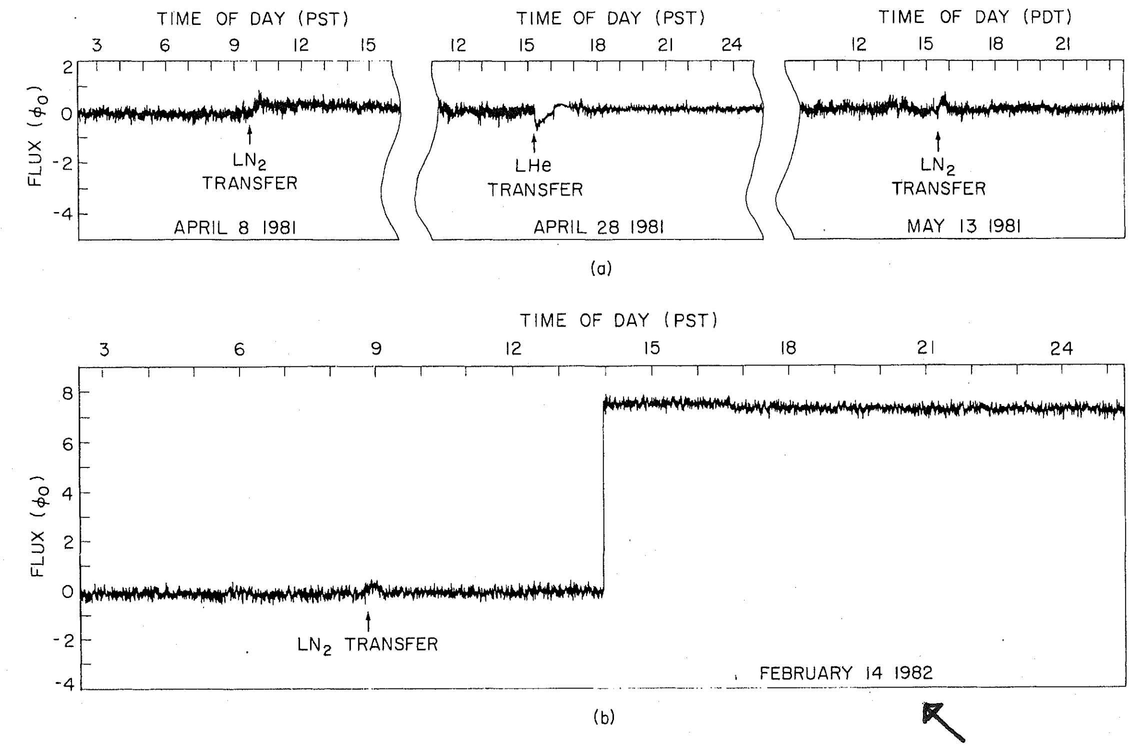


FIG. 2. Data records showing (a) typical stability and (b) the candidate monopole event.

failed to cause detectable offsets.

(b) *rf interference* from the motor brushes of a heat gun failed to produce any offsets when operated in close proximity to the detector.

(c) *External magnetic field* changes are attenuated by 180 dB, primarily from an exponential factor of $e^{-1.83z/a}$, where $z = 72$ cm is the distance in from the open top of the superconducting shield and $a = 10$ cm is the shield radius.

(d) *Ferromagnetic contamination* is minimized using clean-bench assembly techniques and

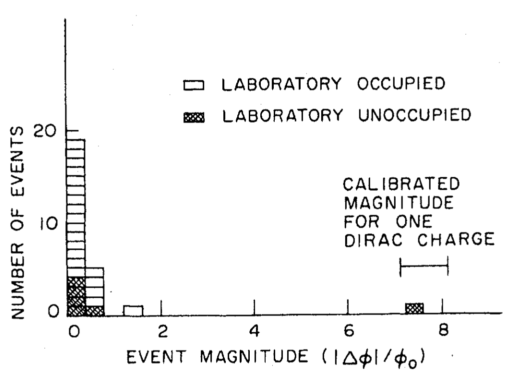


FIG. 3. Histogram of all event magnitudes.

checked with magnetometer measurements within the shield.

(e) *The critical current* of the loop is not reached for currents a thousand times greater than $8\varphi_0/L$ and is typically 10^8 times greater.

(f) *Mechanically induced offsets* have been intentionally generated and are probably caused by shifts of the four-turn loop-wire geometry which produce inductance changes. Sharp raps with a screwdriver handle against the detector assembly cause such offsets. On two occasions out of 25 attempts these have exceeded $6\varphi_0$ (75% of the shift expected from one Dirac charge); however, drifts in the level were seen during the next hour.

(g) *No seismic disturbance* occurred on 14 February 1982.

(h) *Energetic cosmic rays* depositing $\lesssim 1$ GeV/cm in traversing the wire would raise the local wire temperature by only $\lesssim 0.01$ K, but a 5-K change is needed to reach the critical temperature.

A spontaneous and large external mechanical impulse is not seen as a possible cause for the event; however, the evidence presented by this single event does not preclude the possibility of a spontaneous internal stress release mechanism. Regardless, to date the experiment has set an upper limit of 6.1×10^{-10} cm^{-2} sec^{-1} sr^{-1} for the isotropic distribution of any moving particles with magnetic charge greater than 0.06 g.

An observational upper bound on the mass density of monopoles is given by limits on the local "missing mass."[6] Visible matter has a measured local density of $0.09\,M_{\odot}/\mathrm{pc}^3$ (solar masses per cubic parsec), whereas the mass density calculated from the velocity distribution out of the galactic plane is $0.14\,M_{\odot}/\mathrm{pc}^3$. This local "missing mass" density estimate of $0.05\,M_{\odot}/\mathrm{pc}^3$ is in good agreement with the halo mass estimates extrapolated back to our local galactic radius, which give $0.03\,M_{\odot}/\mathrm{pc}^3$. If we assume this entire "hidden mass" to be made up of monopoles of mass 10^{16} GeV/c^2 with isotropic velocities of order 300 km/sec, as suggested from grand unification theories,[7] the number passing through the earth's surface would be 4×10^{-10} cm^{-2} sec^{-1} sr^{-1}.[8] This would result in 1.5 events per year through the detector loop.

The search with the present detector is being continued, and two new systems of larger sensing area are being built.

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[1] L. W. Alvarez, Lawrence Radiation Laboratory Physics Note No. 470, 1963 (unpublished); P. Eberhard, Lawrence Radiation Laboratory Physics Note No. 506, 1964 (unpublished); L. J. Tassie, Nuovo Cimento 38, 1935 (1965); L. Vant-Hull, Phys. Rev. 173, 1412 (1968); P. Eberhard, D. Ross, L. Alvarez, and R. Watt, Phys. Rev. D 4, 3260 (1971).

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[3] Model 330x SQUID system, manufactured by S. H. E. Corp., San Diego, Cal. For theory see J. Clarke, Proc. IEEE 61, 8 (1973).

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[6] For review, see S. M. Faber and J. S. Gallagher, Ann. Rev. Astron. Astrophys. 17, 135 (1979).

[7] J. P. Preskill, Phys. Rev. Lett. 43, 1365 (1979); G. Lazarides, Q. Shafi, and T. F. Walsh, Phys. Lett. 100B, 21 (1981).

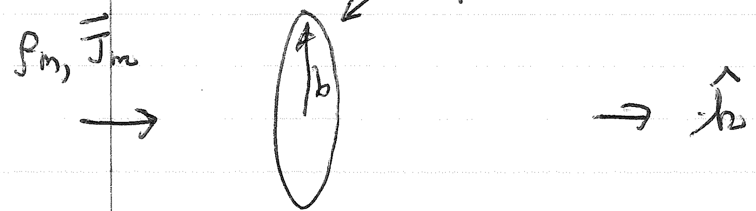
[8] Recently J. D. Ullman, Phys. Rev. Lett. 47, 289 (1981), has reported on the use of a proportional counter array to set an upper flux limit of 3×10^{-11} cm^{-2} sec^{-1} sr^{-1}. However, differing estimates for slow monopole ionization rates [S. Geer and W. G. Scott, CERN $\bar{p}p$ Note 69, April 1981 (unpublished); K. Hayashi, to be published; J. S. Trefil, to be published] indicate the complexities of such calculations. These calculations are crucial to predicting whether slow monopoles are observable in a given ionization detector.

integrate $\vec{\nabla}\times\vec{E}$ over area

$$\oint \vec{E}\cdot d\vec{l} \neq 0 = -\mu_0\int \vec{J}_m\cdot d\vec{a} - \frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$$

Idea

super-conducting ring



Suppose have charge g moving with $\vec{v}$

along axis $\vec{J} = \rho_m \vec{v}$

$$= g\,\delta(\vec{r}-\vec{v}t)\,\mathbf{V}$$

Suppose passes thru center of ring at $\vec{r}=0$, ~~$t=0$~~

Then $$\vec{J} = \frac{g}{\cancel{|\vec{v}|}}\,\delta(t)\,\hat{z}\;\delta(x)\,\delta(y)$$

$$\oint \vec{E}\cdot d\vec{l} + \frac{d\Phi}{dt} = -\frac{\mu_0 g}{\cancel{|v|}}\,\delta(t)$$

$\oint \vec{E}\cdot d\vec{l} = \mathcal{E} = IR$

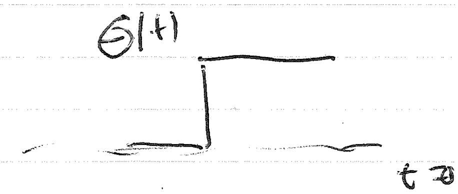
$R\to 0$

$\mathcal{E}=0$

In a superconductor $\oint \vec{E}\cdot d\vec{l} = 0$

$$\frac{d\Phi}{dt} = -\frac{\mu_0 g}{\cancel{|v|}}\,\delta(t), \quad \Phi(t=-\infty)=0$$

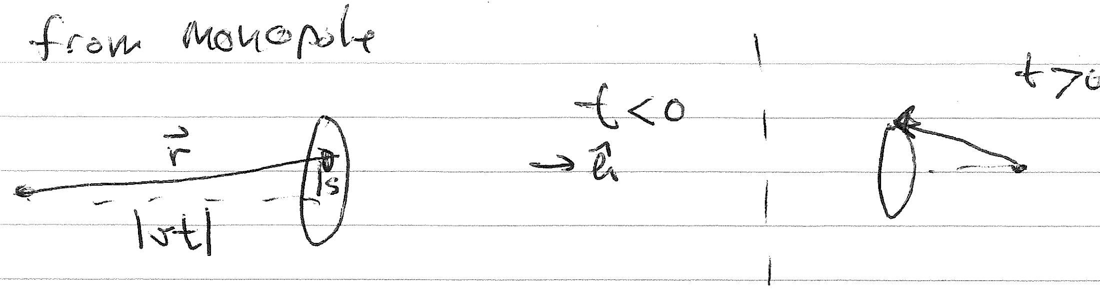
$$\Phi(t) = -\frac{\mu_0 g}{\cancel{|v|}}\,\theta(t)$$



Φ has two contributions

$\Phi = \Phi_g + \Phi_s$ from induced super current

Φ_g from monopole



$$\vec{B} = \frac{g}{4\pi}\frac{\vec{r}}{r^3}\mu_0 \qquad \text{To get flux } \vec{B}\cdot\hat{n}$$

$$t<0 \quad \vec{B}\cdot\hat{n} = \frac{g\mu_0}{4\pi}\frac{v|t|}{(s^2+v^2t^2)^{3/2}}$$

da understood loop $= 2\pi s\,ds$

$$\Phi_g \quad \text{Flux} = \int \vec{B}\cdot\hat{n}\,da = \frac{g}{4\pi}\,v|t|\,2\pi\int_0^b \frac{s\,ds}{(s^2+v^2t^2)^{3/2}}$$

$$= \frac{g\,v|t|}{2}\left[-\frac{1}{(s^2+v^2t^2)^{1/2}}\Big|_0^b\right]\mu_0$$

$$= \frac{g\,v|t|}{2}\mu_0\left[\frac{1}{|vt|} - \frac{1}{(b^2+v^2t^2)^{1/2}}\right]$$

$$= \frac{g\mu_0}{2}\left[1 + \frac{vt}{(s^2+v^2t^2)^{1/2}}\right] \qquad t<0$$

There is an induced supercurrent current I_s in loop

$$\Phi_s = -I(t)\,L \qquad L = \text{self inductance}$$

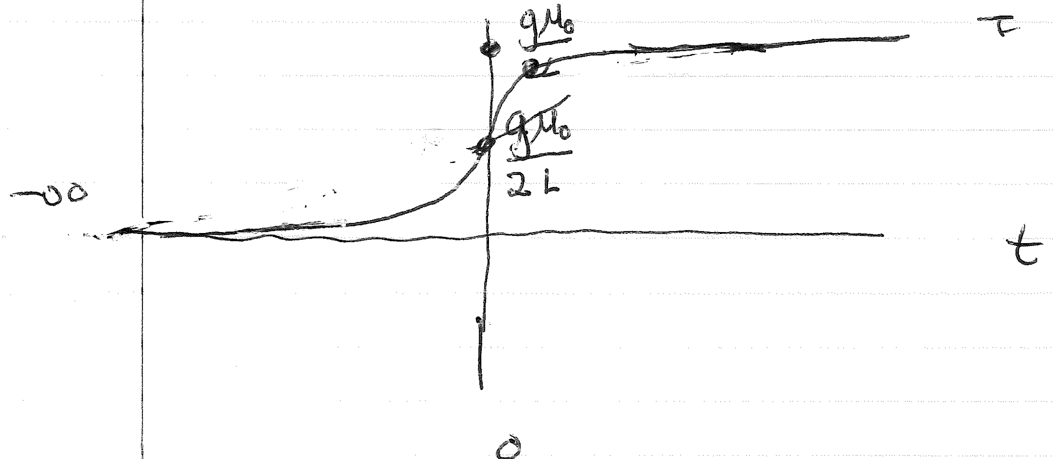
in general $\Phi_g = \frac{g\mu_0}{2}\left[1 - 2\theta(t) + \frac{vt}{(b^2+v^2t^2)^{1/2}}\right]$

$$\Phi(t) \doteq \Phi_s + \Phi_s$$

$$\frac{g\mu_0}{2}\left[1 - 2\theta(t) + \frac{vt}{(b^2+v^2t^2)^{1/2}}\right] - I(t)\,L = -\mu_0 g\,\theta(t)$$

cancel

$$I = \frac{g\mu_0}{2L}\left[1 + \frac{vt}{(b^2+v^2t^2)^{1/2}}\right]$$



Maxwell eqns in Matter

In matter there are bound electric ($\vec{P}$) and magnetic dipoles ($\vec{M}$)

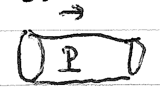
bound current density $\quad \vec{J}_b = \vec{\nabla} \times \vec{M}$

bound charge density $= \rho_b = -\vec{\nabla} \cdot \vec{P}$

Have also written Maxwell eqn in general but now want simplified form involving free charges ρ_f, free currents J_f

* want to use $\vec{H}$ & $\vec{D}$. This done for the static case. Now we have to consider one other effect. This involves polarization – dipole moments/volume

Consider tiny chunk of material that is polarized



There is a surface charge density $\sigma_b = P$ on right and $\sigma_b = -P$ on left (from $\sigma_b = \vec{P} \cdot \hat{n}$)

Now suppose P increases a bit. Then there must be a current from left to right

$$dI = \frac{\partial \sigma_b}{\partial t}\, da_\perp = \frac{\partial P}{\partial t}\, da_\perp$$

In general the current is current density times area

There is a current density

$$\vec{J}_P = \frac{\partial \vec{P}}{\partial t} \qquad \text{Polarization current}$$

This current density is consistent with continuity eq. Recall that $\rho_b = -\vec{\nabla} \cdot \vec{P}$

$$\vec{\nabla} \cdot \vec{J}_P = \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{P} = -\frac{\partial \rho_b}{\partial t}$$ ~~1~~/ES! Includins

$\vec{J}_P$ essential for conservation of bound charge. Now let's write all the sources charges and currents

$$\rho = \rho_f + \rho_b = \rho_f - \vec{\nabla} \cdot \vec{P}$$

$$\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p = \vec{J}_f + \vec{\nabla} \times \vec{M} + \frac{\partial \vec{P}}{\partial t}$$

Gauss Law $\nabla \cdot E = \frac{\rho}{\epsilon_0} = \frac{1}{\epsilon_0} (\rho_f - \vec{\nabla} \cdot \vec{P})$

recall $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho_f}$$

Ampere Maxwell

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} = \mu_0 \left(J_f + \vec{\nabla} \times M + \frac{\partial \vec{P}}{\partial t} \right) + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

$$\vec{\nabla} \times (\mu_0 \vec{H} + \cancel{\vec{M}}) = \mu_0 \left(J_f + \cancel{\vec{\nabla} \times \vec{M}} + \frac{\partial \vec{P}}{\partial t} \right) + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \cancel{\epsilon_0} \frac{\partial(\vec{E}\epsilon_0 + \vec{P})}{\partial t}$$

$$= \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

So $$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}}$$

others as before

$$\left.\begin{aligned} \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \end{aligned}\right] \text{Source free}$$

These are Maxwell eqns in media

To proceed need to know relation between $\vec{D}, \vec{E}$ and $\vec{B}, \vec{H}$ depends on properties of material

For Linear media

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{M} = \mu_0 \chi_m \vec{H}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 (1+\chi_e) \vec{E} = \epsilon \vec{E}$$

$$\vec{B} = \mu_0 (1+\chi_m) \vec{H}$$

$$= \mu \vec{H}$$